APPM 2350 **Exam 2, Wednesday, October 22** **Fall 2008**

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On the front of your bluebook, write: (1) your name, (2) your recitation session number, and draw a grading table, as shown in the margin to the right. Apart from for Problem 1 (for which you should give only the answers), you must show all your work in your blue book, and BOX your final answers (A correct answer with no relevant work shown might not receive any credit, while an incorrect answer with some correct work might receive partial credit). With the exception of a 1 sheet (2 page) handwritten crib sheet, no text book, notes, or calculators are permitted. Please start each new problem on a new page of the bluebook.

1. (40 points) In this multiple choice problem, assume that all unspecified functions are continuously differentiable. For each of the statements below, select option A, B, C, or D:

a. Consider $f(x,y,z)$ where $x = x(u,v)$, $y = y(u)$, $z = z(u,v,w)$. Then $\dfrac{\partial f}{\partial v} =$

A. $\dfrac{\partial f}{\partial x}\cdot\dfrac{\partial v}{\partial x}+\dfrac{\partial f}{\partial y}\cdot\dfrac{\partial v}{\partial y}+\dfrac{\partial f}{\partial z}\cdot\dfrac{\partial v}{\partial z}$

B. $\dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial v}+\dfrac{\partial f}{\partial z}\cdot\dfrac{\partial z}{\partial v}$

C. $\dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial y}\cdot\dfrac{\partial y}{\partial v}+\dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial z}\cdot\dfrac{\partial z}{\partial v}+\dfrac{\partial f}{\partial z}\cdot\dfrac{\partial z}{\partial x}\cdot\dfrac{\partial x}{\partial v}$

D. None of the above

b. How many critical points does the function $f(x,y,z) = \cos x + y e^x + xz$ have?

A. One B. Two C. None D. Infinitely many

c. Which of the following vectors is normal to the surface $z = f(x,y)$?

A. ∇f B. $f_x\,\mathbf{i} + f_y\,\mathbf{j}$ C. $f_x\,\mathbf{i} + f_y\,\mathbf{j} - \mathbf{k}$ D. None of these

d. What is the value of $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$?

A. 0 B. 1 C. Infinite D. Does not exist

e. $\displaystyle\int_0^1\int_{\sqrt{y}}^1 f(x,y)\,dx\,dy =$

A. $\displaystyle\int_{\sqrt{y}}^1\int_0^1 f(x,y)\,dy\,dx$ B. $\displaystyle\int_0^1\int_0^{x^2} f(x,y)\,dy\,dx$ C. $\displaystyle\int_0^1\int_{x^2}^1 f(x,y)\,dy\,dx$ D. $\displaystyle\int_0^1\int_0^{\sqrt{y}} f(x,y)\,dy\,dx$

f. The directional derivative of a function $f(x,y,z)$ is largest in which direction?

A. ∇f B. $-\nabla f$ C. Orthogonal to ∇f D. $x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$

g. The linearization of $f(x,y) = x^2 + y^2 + 1$ around $(1,1)$ is

A. 1 B. 3 C. $3 + 2x + 2y$ D. $-1 + 2x + 2y$

Please turn over ⇒

h. If the variables x and y (not equal to zero) are both uncertain by 1% , then their product $x\,y$ is uncertain by about

A. 1% B. 2% C. 0.01% D. Can't tell

i. If the variables x and y (not equal to zero; also $x \neq -y$) are both uncertain by 1% , then their sum $x + y$ is uncertain by about

A. 1% B. 2% C. 0.01% D. Can't tell

j. On the curve $x = y^2$, the function $f(x,y) = 3y - x$ has

A. a global minimum only B. a global maximum only

C. a saddle point D. both a global maximum and minimum

2. (20 points) Find all critical points of $f(x,y) = \frac{1}{3}x^3 + xy^2 - x$ and classify each critical point as a local maximum, local minimum, or saddle point.

3. (20 points) Consider finding the global minimum of $f(x,y,z) = x^2 + (y+z)^2$ on the plane $x = 1 - 2y$.

a. Use the method of Lagrange's multipliers to set up a nonlinear system of equations.

b. Find all possible solutions to the system in part (a).

c. Determine the (x,y,z)- location and the value of the minimizing point.

4. (20 points) The volume of the region $R = \left\{ \sqrt{1-x^2} \le y \le 2\sqrt{1-x^2}\ ,\ 0 \le z \le y \right\}$ can be found by multivariate integration.

a. Sketch the domain $D = \left\{ \sqrt{1-x^2} \le y \le 2\sqrt{1-x^2} \right\}$ in the x,y- plane. Shade the interior of the domain and clearly mark the points where the curves that bound the domain intersect.

b. Find the volume of R by a suitable integration over the domain D.