APPM 2350 **Exam 3, Wednesday, November 19** **Fall 2008**

On the front of your bluebook, write: (1) your name, (2) your recitation session number, and draw a grading table, as shown in the margin to the right. Apart from for Problem 1 (for which you should give only the answers), you must show all your work in your blue book, and BOX your final answers (A correct answer with no relevant work shown might not receive any credit, while an incorrect answer with some correct work might receive partial credit). With the exception of a 1 sheet (2 page) handwritten crib sheet, no text book, notes, or calculators are permitted. Please start each new problem on a new page of the bluebook.

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1. (24 points) In this multiple choice problem, assume that all unspecified functions are continuously differentiable. For each of the statements below, select option A, B, C, or D:

a. The value of $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$ is

A. π B. 2π C. $\frac{4\pi}{3}$ D. 6π

Hint: The problem can be solved quickly without working out the triple integrals.

b. The center of mass of a uniform density half-circle is located how far (in terms of the radius) from the object's straight edge?

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{4}{3\pi}$ D. $\frac{\pi}{4}$

c. The Taylor approximation around the origin to a function $f(x,y,z)$ of three variables, up to and including linear terms, contains (at most) how many terms?

A. 2 B. 3 C. 4 D. 5

d. What object is described by the spherical coordinate relation $\rho = \cos\phi,\ 0 \le \phi \le \frac{\pi}{2}$?

A. A plane B. A circle C. A sphere D. None of the above.

e. The formulas to convert from spherical to cylindrical coordinates are

A. $\begin{cases} r = \rho\sin\phi \\ \theta = \theta \\ z = \rho\cos\phi \end{cases}$ B. $\begin{cases} r = \rho\cos\phi \\ \theta = \theta \\ z = \rho\sin\phi \end{cases}$ C. $\begin{cases} r = \rho\sin\phi\cos\theta \\ \theta = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$ D. None of the above.

f. One of the following expressions for the work integral is NOT correct. Identify which one that is.

A. $\int \boldsymbol{F}\cdot\boldsymbol{T}\,ds$ B. $\int \boldsymbol{F}\cdot d\boldsymbol{r}$ C. $\int \boldsymbol{F}\cdot\boldsymbol{n}\,ds$ D. $\int \boldsymbol{F}\cdot\frac{d\boldsymbol{r}}{dt}\,dt$

g. The volume of a pyramid can be expressed in terms of its base area A and its height h as

A. $V = Ah$ B. $V = \frac{1}{2}Ah$ C. $V = \frac{1}{3}Ah$ D. $V = \frac{1}{4}Ah$

h. The vector field $\boldsymbol{F} = (z+y)\,\boldsymbol{i} + x\,\boldsymbol{j} + g(x,y,z)\,\boldsymbol{k}$ becomes conservative if we choose $g(x,y,z)$ as

A. $x+y$ B. x^2 C. 2 D. $x+\sin^2 z$

Please turn over ⟹

2. (25 points) Consider the following triple integral of $f(x,y,z)$ in cylindrical coordinates:

$$I = \int_0^{2\pi} \int_0^1 \int_0^1 f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta \, .$$

Set up this integration in different coordinate systems ond orders of integration as specified below.
DO NOT EVALUATE ANY INTEGRAL IN THIS PROBLEM!

a. Draw the region of integration, D, in xyz space. Briefly describe D in words.
<u>Hint:</u> Make sure you have this part correct before you move on.

b. Rewrite I in $dr\, dz\, d\theta$ ordering.

c. Rewrite I in $dx\, dy\, dz$ ordering.

d. Rewrite I in $d\rho\, d\phi\, d\theta$ ordering.

e. Rewrite I in $d\phi\, d\rho\, d\theta$ ordering.

3. (25 points) Let $I = \iint_{R_0} x^2 \cos x^2 \, dx\, dy$, where $R_0 = \{1 \le x \le 2,\ \frac{2}{x} \le y \le \frac{3}{x}\}$. Consider substituting x and y with new variables

$$u = x^2 \qquad \text{and} \qquad v = xy.$$

a. Write x and y in terms of the new variables.

b. Draw R_0 in the xy-plane. Clearly label each curve and intersection.

c. Draw R_1 (the image of R_0) in the uv-plane. Again clearly label each curve and intersection.

d. Set up I as a double integral in the new variables.

e. Evaluate the integral from part d. to find I.

4. (26 points) Consider the half-circle $R = \{y \ge -x,\ x^2 + y^2 \le 2\}$ and the curve C that traces the boundary of R from $(1,-1)$ counter-clockwise along the circular edge to $(-1,1)$ and then back to $(1,-1)$ along the straight edge.

a. Find an appropriate parametrization for C. <u>Hint:</u> Check your work!

b. Let $F(x,y) = x\,\hat{\boldsymbol{i}} + y\,\hat{\boldsymbol{j}}$. Find the circulation, $\int_C \boldsymbol{F}\cdot\boldsymbol{T}\, ds = \int_C M\,dx + N\,dy$.

c. Using the same field as for part b, find the outward flux $\int_C \boldsymbol{F}\cdot\boldsymbol{n}\, ds = \int_C M\,dy - N\,dx$.

d. Evaluate $\iint_R e^{x^2+y^2}\, dx\, dy$.