APPM 2350 **<u>Final</u>, Monday, December 15** **Fall 2008**

On the front of your bluebook, write: (1) your name, (2) your recitation session number, and draw a grading table, as shown in the margin to the right. Apart from for Problem 1 (for which you should give only the answers), you must show all your work in your blue book, and BOX your final answers (A correct answer with no relevant work shown might not receive any credit, while an incorrect answer with some correct work might receive partial credit). With the exception of a 1 sheet (2 page) handwritten crib sheet, no text book, notes, or calculators are permitted. Please start each new problem on a new page of the bluebook.

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1. (60 points) In this multiple choice problem, assume that all unspecified functions are continuously differentiable. For each of the statements below, select option A, B, C, or D:

a. The slope of the normal to the line $ax+by=c$ is
A. $\frac{a}{b}$ B. $-\frac{a}{b}$ C. $\frac{b}{a}$ D. $-\frac{b}{a}$

b. The area of a parallelogram with vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ forming two of the sides is
A. $|\boldsymbol{A}\cdot\boldsymbol{B}|$ B. $|\boldsymbol{A}\times\boldsymbol{B}|$ C. $|\boldsymbol{A}\cdot\boldsymbol{A}+\boldsymbol{B}\cdot\boldsymbol{B}|$ D. None of the above

c. Which of the following formulas describes the length of the periphery of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$?
A. $\pi(a+b)$ B. $2\pi\sqrt{ab}$
C. $\int_0^{2\pi}\sqrt{a^2\cos^2 t+b^2\sin^2 t}\,dt$ D. None of the above

d. What is the value of $\lim_{(x,y)\to(0,0)}\frac{xy}{x+y}$?
A. 0 B. 1 C. ∞ D. Does not exist

e. Let $f(x,y)=x^2+y^2$ with $x=\sin(t^2)$, $y=\cos(t^2)$. If t is uncertain by 1%, how uncertain is f?
A. 0% B. 1% C. π% D. Can't tell

f. What is the linearization of $\sqrt{xyz}$ around (1,1,1)?
A. $\frac{1}{3}(x+y+z)$ B. $\frac{1}{4}(1+x+y+z)$ C. $-\frac{1}{2}(1-x-y-z)$ D. None of the above

g. In which direction (measured at the origin as angle θ from the x-axis) does the function $f(x,y)=(\tan x)(\cos y)$ decrease the fastest?
A. 0 B. $\pi/3$ C. $\pi/2$ D. π

h. $\int_0^1\int_{1-y}^1 f(x,y)dxdy=$
A. $\int_0^1\int_x^1 f(x,y)dydx$ B. $\int_0^1\int_1^{1-x} f(x,y)dydx$ C. $\int_0^1\int_0^1 f(x,y)dydx$ D. $\int_0^1\int_{1-x}^1 f(x,y)dydx$

i. The point $(x,y,z)=(0,1,1)$ in rectangular coordinates corresponds in spherical (ρ,ϕ,θ) coordinates to
A. $(\sqrt{2},\frac{\pi}{4},\frac{\pi}{2})$ B. $(1,\frac{\pi}{4},\frac{\pi}{2})$ C. $(\sqrt{2},\frac{\pi}{4},0)$ D. None of the above

j. Which of the following 2-D vector fields is conservative
A. $\boldsymbol{F}=xy\,\boldsymbol{i}+xy\,\boldsymbol{j}$ B. $\boldsymbol{F}=xy^2\,\boldsymbol{i}+x^2y\,\boldsymbol{j}$ C. $\boldsymbol{F}=x\,\boldsymbol{i}+x\,\boldsymbol{j}$ D. $\boldsymbol{F}=x^2y\,\boldsymbol{i}+xy^2\,\boldsymbol{j}$

Please turn over $\Longrightarrow$

On problems 2-5, anywhere you use **Green's Theorem**, **Stokes' Theorem**, or the **Divergence Theorem**, you must write the name of the theorem and draw a box around it for full credit.

2. (35 points) Consider the curve $C:\ \{x(t)=t^2,\ y(t)=t^3-t,\ -1\le t\le 1\}$.

   a. Give equations for lines tangent and normal to C when $t=1/2$.

   b. What are the minimum and maximum values of $f(x,y)=x+y$ on C? Give the (x, y) locations where these values are attained.
   <u>Hint:</u> No Lagrange multipliers are necessary for this problem.

   c. Set up and evaluate an integral to find the area of R, the region enclosed in C.

3. (35 points) Consider the field $\boldsymbol{F}(x,y,z)=-y\,\boldsymbol{i}+x\,\boldsymbol{j}+\boldsymbol{k}$. Let S be the bell-shaped surface $x^2+y^2+(z-1)\,e^{xyz}=1$ that is above the plane $z=1$. Let D be the region of space between S and $z=1$.
   <u>Hint:</u> Both parts a and b below can be solved with relatively little work.

   a. Calculate the surface-curl integral $\iint_S (\nabla\times\boldsymbol{F})\cdot\boldsymbol{n}\,d\sigma$.

   b. What is the outward flux through S?

4. (35 points) Scott jumped the Royal Gorge with a jet engine on his back along an unknown path, $\boldsymbol{r}(t)=x(t)\,\boldsymbol{i}+y(t)\,\boldsymbol{j}+z(t)\,\boldsymbol{k}$. The power output, $P(g(t),T(x(t),y(t),z(t)),z(t))$, of his engine is a function of gas consumption, g, temperature, T, and altitude, z. In turn, g is a function of time, t, and T is a function of space, (x,y,z). At a certain time, t_1, Scott's equipment gives him the following measurements:

   Temperature Gradient: $\nabla T(t_1)=\boldsymbol{j}+\boldsymbol{2k}$ Velocity: $v(t_1)=\boldsymbol{3\,i}+\boldsymbol{4\,j}$
   Consumption Change: $g'(t_1)=5$
   Power Changes: $\dfrac{\partial P}{\partial g}(t_1)=6,\quad \dfrac{\partial P}{\partial T}(t_1)=0,\quad \dfrac{\partial P}{\partial z}(t_1)=7,$

   a. Use the chain rule to write a general formula for dP/dt. Then, evaluate the change in power output with respect to time at t_1.

   b. What is the change in power output with respect to change in distance at t_1?

   c. Give a vector, **u**, that has the direction of greatest temperature decrease at t_1. Also at t_1, give another vector, **w**, with direction that is locally a direction of no temperature change.
   <u>Note:</u> You can give vectors of any magnitude.

5. (35 points) Consider the function $f(x,y)=2xy+1$ on the unit circle, $R:\ x^2+y^2\le 1$.

   a. Find and classify all critical points of $f(x,y)$ in the interior of R, $x^2+y^2<1$.

   b. Use the method of Lagrange's multipliers to find any critical points of $f(x,y)$ on the boundary of R, $x^2+y^2=1$.
   <u>Hint:</u> there are four critical points.

   c. Give the global minimum and maximum of $f(x,y)$ in R and all points in R where these values are attained.

   d. Find the surface area of $z=2xy+1$ above R.
   <u>Hint:</u> polar coordinates are useful for the final integral evaluation.