RETURN THIS PROBLEM SET WITH YOUR SOLUTIONS

APPM 2350 **Final, Sunday, December 14** **Fall 2008**

On the front of your bluebook, write: (1) your name, (2) your recitation session number, and draw a grading table, as shown in the margin to the right. Apart from for Problem 1 (for which you should give only the answers), you must show all your work in your blue book, and BOX your final answers (A correct answer with no relevant work shown might not receive any credit, while an incorrect answer with some correct work might receive partial credit). With the exception of a 1 sheet (2 page) handwritten crib sheet, no text book, notes, or calculators are permitted. Please start each new problem on a new page of the bluebook.

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1. (60 points) In this multiple choice problem, assume that all unspecified functions are continuously differentiable. For each of the statements below, select option A, B, C, or D:

a. What type of surface is described by the equation $(x^2+y^2+z^2)+2(x+y+z)=0$?
A. Origin (0,0,0) only B. Sphere with radius 1
C. Sphere with radius less than 1 D. None of the above.

b. The distance from the origin to the plane $x+y+z=1$ is
A. 1 B. 1/3 C. $1/\sqrt{3}$ D. 1/6

c. What is the value of $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{\sqrt{x}-y}$?
A. 0 B. 1 C. ∞ D. Does not exist

d. If the radius of a sphere is uncertain by about 1%, how uncertain is then the volume?
A. 1% B. 3% C. π% D. Can't tell

e. If $f(x,y(x))=0$, then $\frac{dy}{dx}=$
A. $\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}$ B. $-\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}$ C. $\frac{\partial f}{\partial x} \times \frac{\partial f}{\partial y}$ D. None of the above

f. In the $\boldsymbol{T}\,\boldsymbol{N}\,\boldsymbol{B}$ system, it holds that
A. $\boldsymbol{B}\times\boldsymbol{T}=\boldsymbol{N}$ B. $\boldsymbol{B}\times\boldsymbol{N}=\boldsymbol{T}$ C. $\boldsymbol{T}\times\boldsymbol{B}=\boldsymbol{N}$ D. $\boldsymbol{T}\times\boldsymbol{N}=-\boldsymbol{B}$

g. $\int_0^1\int_y^1 f(x,y)\,dx\,dy=$
A. $\int_0^1\int_x^1 f(x,y)\,dy\,dx$ B. $\int_0^1\int_1^x f(x,y)\,dy\,dx$ C. $\int_0^1\int_0^1 f(x,y)\,dy\,dx$ D. $\int_0^1\int_0^x f(x,y)\,dy\,dx$

h. When converting a triple integral from rectangular (x,y,z) to spherical (ρ,ϕ,θ) coordinates, we need to insert a factor of
A. $\rho\sin\theta$ B. $\rho^2\sin\theta$ C. ρ D. $\rho^2\sin\phi$

i. The theorem that relates the work (flow) integral around a closed 3-D curve to a certain surface integral is known as
A. Divergence theorem B. Green's theorem
C. Stokes' theorem D. Gradient theorem

j. Let $F=\nabla f$ where $f=x^2+y^2+z^2$. What is the value of $\int_{(0,0,0)}^{(1,1,1)} \boldsymbol{F}\cdot d\boldsymbol{r}$?
A. 1 B. 2 C. 3 D. Can't tell

Please turn over ⇒

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On problems 2-5, anywhere you use **Green's Theorem**, **Stoke's Theorem**, or the **Divergence Theorem**, you must write the name of the theorem and draw a box around it for full credit.

2. (35 points) Consider the curve $C: \{x(t) = t - t^2,\ y(t) = t^2 - t^3,\ 0 \le t \le 1\}$.

   a. Give equations for lines tangent and normal to C when $t = 1/3$.

   b. What are the minimum and maximum values of $f(x, y) = x + y$ on C?
   Hint: No Lagrange multipliers are necessary for this problem.

   c. Set up and evaluate an integral to find the area of R, the region enclosed in C.

3. (35 points) Hint: Both parts a and b below can be solved with relatively little work.

   a. Consider $\boldsymbol{F} = -xy\,\boldsymbol{i} + xy\,\boldsymbol{j} + (zx - zy)\,\boldsymbol{k}$. Calculate the surface-curl integral $\iint_{S_1} (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{n}\, d\sigma$ where S_1 is the bell-shaped surface $x^2 + y^2 + z/(1 + x^2 + y^2) = 1$ that lies above $z = 0$.

   b. Find the outward flux of $\boldsymbol{F} = xz\,\boldsymbol{i} + yz\,\boldsymbol{j} + e^{(y^2)} \cos x^2\, \boldsymbol{k}$ through the six-sided surface S:
   $\{-1 \le x \le 1,\ -1 \le y \le 1, 0 \le z \le \sqrt{4 - x^2 - y^2}\ \}$.

4. (35 points) A climber walks on the surface of a glacier described by $f(x, y) = y^2 - yx^2 + y$.

   a. Find all three critical points of $f(x, y)$ for the climber. Classify each of these points as a local minimum, local maximum, or saddle point.

   b. At the point $P(2,1)$, give a two-dimensional vector, **u**, that has the direction of steepest descent (the direction of greatest decrease of $f(x, y)$).

   c. The climber walks on an unknown path, $\boldsymbol{r}(t) = x(t)\,\boldsymbol{i} + y(t)\,\boldsymbol{j}$, through point P at time t_1. If the climber' velocity at t_1 is $\boldsymbol{v}(t_1) = 5\,\boldsymbol{i} - 6\,\boldsymbol{j}$, what is the change of $f(x, y)$ with respect to time, $\frac{df}{dt}$, that the climber experiences?

5. (35 points) Assume the satellite dish for your digital television is represented by the surface of a parabaloid, $S: \{x = y^2 + z^2,\ x \le 1\}$. While you wait all day for the cable guy, you need to make the following calculations:

   a. Find the surface area of the dish.
   Hint: a transformation to polar coordinates is useful for the final integration.

   b. Setup, **but do not evaluate**, an integral for the volume of the region of space inside the dish, D: $\{y^2 + z^2 \le x \le 1\}$, in $dy\, dz\, dx$ ordering. (Any other ordering will only receive half credit).

   c. Use the method of Lagrange multipliers to find the point on the dish that minimizes the function $x^2 + y^2 - z/2$.