1. (35 points) Find the absolute maximum and minimum of $f(x, y) = 20 - 4x - 16y + x^2 + 4y^2$ on the closed triangular region plate bounded by the lines $x = 0$, $y = x$ and $y = 4$. Be sure to clearly give both the locations and values of the absolute extremum.

2. (30 points) Consider the integral $I = \iint_R \left(\sqrt{\frac{y}{x}} + 3y \right) dx\, dy$ where R is the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy = 1$, $xy = 9$ and the lines $y = x$ and $y = 4x$.

   (a) Carefully sketch the region R in the xy-plane.

   (b) Using the transformation $x = u/v$ and $y = uv$, with $u > 0$ and $v > 0$, graph the region in the uv-plane that corresponds to R.

   (c) Rewrite I as an integral over an appropriate region G in the uv-plane.

   (d) Evaluate the integral I in the uv-plane.

3. (22 points) Consider the following integral: $$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{x^2+y^2}} dz\, dy\, dx.$$

   (a) Sketch the region that is being integrated.

   (b) Convert the original integral into cylindrical coordinates using the order $dr\, dz\, d\theta$ but **do not solve**.

   (c) Convert the original integral into spherical coordinates using the order $d\rho\, d\phi\, d\theta$ but **do not solve**.

   (d) Solve **one** of the integrals.

4. (18 points) Consider the region R in the xy-plane bounded by $y = x + 1$, $y = x - 1$, $y = -x + 5$, and $y = -x + 3$.

   (a) Sketch the region.

   (b) Use the transformations $x = \frac{1}{2}(u + v)$ and $y = \frac{1}{2}(u - v)$ to sketch the new region S in the uv-plane.

   (c) If the density of the region R is given by $f(x, y) = e^{(x+y)}$, find the mass of the region by using the transformations given in (b).

5. (22 points) Consider the curve $w = x^2 y^3 z^4$ where x, y, and z are defined in terms of the line $x = 2t + 1$, $y = 3t + 2$, and $z = 5t + 4$ for $t \geq 0$.

   (a) Find $\frac{dw}{dt}$.

   (b) What is the distance between the origin and the specified line?

   (c) The vector pointing parallel to the specified line is perpendicular to a plane that contains the origin. Find the equation for this plane.

   (d) What is the directional derivative of the curve w at the point $(1, -1, 1)$ in the direction of a vector pointing along the line?

6. (25 points) Suppose that C is the curve defined by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sqrt{2}\sin t\,\mathbf{j} + \cos t\,\mathbf{k}$, $0 \leq t \leq 2\pi$.

(a) Find the unit tangent vector to C at the point $(0, 1, 0)$.

(b) Find the principle unit normal vector to C at the point $(0, 1, 0)$.

(c) Find the curvature κ of C at the point $(0, 1, 0)$.

(d) Find the length of the curve C.

7. (30 points) Suppose that a basketball (defined by the surface $x^2 + y^2 + z^2 = 1$) is put in a microwave oven (can you think of a better use for one?) and heated to the temperature $T = 900xyz^2$. Find the point(s) on the ball with the hottest temperature. The ball will explode if the temperature at a point on the surface exceeds 150^o, the melting temperature of its plastic. Does it?

8. (30 points) You have just purchased a rather large section of intergalactic space. Relative to an intergalactic Cartesian coordinate system, your section of space is given by $1 \leq x \leq 2$, $0 \leq xy \leq 2$ (note: that's an xy in there!), and $0 \leq z \leq 1$, where distances are measured in parsecs. In your section of space the density of interstellar matter is given by $x^2y + 3xyz$. You now want to know just how much interstellar matter you know own.

(a) Clearly sketch your region of space in the xyz-coordinate system.

(b) Write out in detail, but do not evaluate, the integral that will allow you to determine just how much interstellar matter you own.

(c) To do the volume integration, you decide to use the transformation $u = x$, $v = xy$ and $w = 3z$. Clearly sketch your region of space in the uvw-coordinate system.

(d) Rewrite the integral in part (b) terms of u, v and w and then evaluate it.

9. A rectangular box without a lid is to be made from $12\,m^2$ of cardboard. Find the max volume of such a box using Lagrange multipliers.