APPM 2350 CALC III REVIEW EXAM#2 NAME…………………………………

CALCULUS III

PART A :

12.1 Functions of Several Variables,
12.2 Limits and Continuity

1. True or false

(a) Any non-empty region (subset) of the plane is either bounded or unbounded.

2. Let $g(x,y)=\dfrac{1}{\sqrt{x^2+4y^2-1}}$.

(a) Find the domain and range of g.

(b) Carefully sketch the level curve through (0,1) and find a simple equation for it.

3. Find the domain and range of $f(x,y)=\dfrac{3}{\sqrt{x^2-y}}$.

4. Let $f(x,y)=\sqrt{x^2-y^2}$.

(a) What is the domain of $f(x,y)$? Sketch it.

(b) Describe the domain in terms of open/closedness and boundedness.

(c) Is $f(x,y)$ continuous at $(x,y)=(0,0)$?

(d) The vector $\mathbf{n}=-5\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ is normal to the surface $z=f(x,y)$ at $(x,y)=(5,3)$. Find the equation of the plane that approximates f near $(5,3)$ -i.e. the plane that has the same normal and goes through same point.

5. Consider the function $f(x,y)=\dfrac{x^2\sqrt{1-xy}}{x^2+y^2}$.

(a) What is the domain of $f(x,y)$? Sketch it; be sure to show which boundary points are included in the domain and which are not.

(b) Describe the domain of f in terms of open/closedness and boundedness.

(c) Find the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the y-axis.

(d) Find the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the line $y=x$.

(e) What, if anything, do your answer to (c) and (d) tell you about $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

(f) Is $f(x,y)$ continuous at $(0,0)$?

6. Consider the function $f(x,y)=\dfrac{x\sqrt{1-(x^2+y^2)}}{x-y}$.

(a) What is the domain of $f(x,y)$? Sketch it.

(b) Describe the domain of f in terms of open/closedness and boundedness.

(c) Find the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the x-axis.

(d) Find the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the y-axis.

(e) What, if anything, do your answer to (c) and (d) tell you about $\lim_{(x,y)\to(0,0)} f(x,y)$?

(f) Is $f(x,y)$ continuous at $(0,0)$?

7. Consider the function $f(x,y) = x\sqrt{\frac{1}{x^2+y^2}-1}$.

(a) What are the domain and range of $f(x,y)$? Include a sketch of the domain.

(b) Is the domain of f open or closed? Is it bounded? Give reasons for your answer.

(c) Determine $\lim_{(x,y)\to(0,a)} f(x,y)$, where $0<a<1$ is a constant.

(d) Is $f(x,y)$ continuous at the point $(0,a)$ (again, where $0<a<1$)? Give reasons for your answer.

(e) Determine $\lim_{y\to 0} f(0,y)$.

(f) Determine $\lim_{x\to 0} f(x,0)$.

(g) What, if anything, do your answer to (e) and (f) tell you about $\lim_{(x,y)\to(0,0)} f(x,y)$?

(h) Is $f(x,y)$ continuous at the point $(0,0)$? Give reasons for your answer.

8. Let $z = f(x,y)$ be given by $f(x,y) = 4x^2 + y^2 - 4$.

(a) Find the domain and range of f.

(b) Make a sketch of graph of this function. On your sketch, label the axes and indicate the level curve for $z=0$.

(c) What is the name of the surface sketched in part (b)?

9. Determine the domain and range of the function $f(x,y) = \frac{1}{\sqrt{16-x^2-y^2}}$. Describe the level curves of $f(x,y)$ and sketch one level curve. Justify your answer.

10. Determine (and sketch) the domain D and give the range R for the function $f(x,y) = \sum_{k=0}^{\infty}(x^2-y^2)^k$. Specify in particular if the boundaries of the region/interval are included or not.

11. State the domain and range of the function $F(x,y,z) = (1-x^2)^{1/2} e^{(y^2+z^2)}$.

12. Consider the function $f(x,y) = \sin^{-1}(xy)$.

(a) Find the function's domain.

(b) Give the inequalities that describe the function's domain. Draw a picture and shade the region in the x-y plane.

(c) Explain whether the domain is bounded or unbounded, and closed or open.

(d) Find the function's range.

(e) Explain whether the range is bounded or unbounded, and closed or open.

13. Consider the function $z = \cos(x+y)$.

(a) Find its domain and range of $z=\cos(x+y)$.

(b) Plot the level set for $z=\tan(x^2+4y^2)$ when $z=1$. Show a much detail as possible.

(c) Write down the linearization for $z=x^2\sqrt{y}$ at $(3,-1)$.

(d) Describe the level sets of the $z=f(x-y)$ for f a continuous function.

14. Consider the function $f(x,y)=\dfrac{\sin(xy)}{xy}$.

(a) What is the domain of $f(x,y)$?

(b) What is $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

(c) If $\left(1-\dfrac{x^2y^2}{6}\right)<f(x,y)<1,$ how could one define $f(0,0)$ to make $f(x,y)$ continuous at the origin?

15. Consider the function $f(x,y)=\sin\sqrt{\dfrac{\pi^2}{4}-x^2-y^2}$.

(a) What is the domain of $f(x,y)$?

(b) Is the domain open, closed, or neither? Support your answer with a brief explanation.

(c) Is the domain bounded or unbounded? Support your answer with a brief explanation.

(d) What is the range of $f(x,y)$?

16. Let $f(x,y)=\begin{cases}\dfrac{\sin(x-y)}{|x|+|y|}, & |x|+|y|\neq 0\\ 0 & ,\ (x,y)=(0,0)\end{cases}$. Is f continuous at the origin? Why?

17. Let $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$ for $(x,y)\neq(0,0)$. Is it possible to define $f(0,0)$ in a way that makes f continuous at origin? Why?

18. Consider the functions

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0 & ,\ (x,y)=(0,0)\end{cases}\quad \text{and}\quad g(x,y)=e^{4x}(\cos y+\sin y).$$

(a) Compute $f_x(0,0)$ and $f_y(0,0)$.

(b) Is $f(x,y)$ continuous at $(0,0)$? Give reasons for your answer.

(c) Find the linearization of $g(x,y)$ at $(0,0)$.

(d) Around the point $(0,0)$, is $g(x,y)$ more sensitive to change in x, or to change in y? Give reasons for your answer.

19. Find the equation for the level curve of the function $f(x,y)=\int_x^y \frac{dt}{1+t^2}$ that passes through the point $(-\sqrt{2},\sqrt{2})$.

20. Consider the function $g(x,y,z)=\sqrt{x^2+y^2+z^2}$. Find a simple equation for the level surface through the point (-1,0,3). Sketch/describe this particular level surface.

21. Evaluate the limit or explain why the limit does not exist.

(a) $\lim\limits_{(x,y)\to(1,1)} \frac{x^2-y^2}{x+2y}$ (b) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{|xy|}$ (c) $\lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$ (d) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

(e) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ (f) $\lim\limits_{(x,y)\to(2,-2)} \frac{x^2-y^2}{x+y}$ (g) $\lim\limits_{(x,y)\to(0,0)} \frac{x+y}{x-y}$ (h) $\lim\limits_{(x,y)\to(0,0)} \frac{x-y}{x+y}$

(i) $\lim\limits_{(x,y)\to(1,1)} \frac{x-y}{\sqrt{x}-\sqrt{y}}$ (j) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$ (k) $\lim\limits_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^4}$ (l) $\lim\limits_{(x,y)\to(1,1)} \frac{xy+x-y-1}{x-1}$

(m) $\lim\limits_{(x,y)\to(\sqrt{2},\sqrt{2})} \frac{x^3y^3-8}{xy-2}$ (n) $\lim\limits_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ (o) $\lim\limits_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x}$

(p) $\lim\limits_{(x,y)\to(0,0)} \frac{e^{(y-1)}\sin x}{x}$ (q) $\lim\limits_{(x,y)\to(0,0)} \frac{x\sin y}{e^y-1}$ (r) $\lim\limits_{(x,y)\to(0,0)} \frac{2x^4y}{x^8+4y^2}$

PART B:
12.3 Partial Derivatives, 12.5 The Chain Rule

1. True or false

(a) If $\frac{\partial}{\partial s}\left(\frac{\partial g}{\partial t}\right)=\frac{t^2\cos(st)}{s^2+6}$, then $\frac{\partial}{\partial t}\left(\frac{\partial g}{\partial s}\right)=\frac{t^2\cos(st)}{s^2+6}$.

(b) If $p=p(a,b,c,t)$ and $a=a(x,y), b=b(x,t), c=c(y,t)$, then $\frac{\partial p}{\partial t}=\frac{\partial p}{\partial b}\frac{\partial b}{\partial t}+\frac{\partial p}{\partial c}\frac{\partial c}{\partial t}+\frac{\partial p}{\partial t}$.

(c) If f = f(x,y) where x = x(u,v), y = y(u), and u = u(t), v = v(t), then $\frac{df}{dt}=\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial f}{\partial y}\frac{dy}{du}\right)\frac{du}{dt}+\frac{\partial f}{\partial x}\frac{\partial x}{\partial v}\frac{dv}{dt}$.

(d) If $f=f(x(t),y(t),z(t))$, then $\frac{df}{dt}=\frac{\partial f}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial f}{\partial z}\cdot\frac{dz}{dt}$.

(e) Consider f which is a function of t and s, which are both functions of x and y. Then $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial t}\frac{\partial t}{\partial x}+\frac{\partial f}{\partial s}\frac{\partial s}{\partial x}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial x}$.

(f) If f and all its partial derivatives are continuous everywhere, then (in general) f has five distinct fourth-order partial derivatives.

2. If f(u,v,w) is differentiable and u = x-y, v = y-z, w = z-x, then show that $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=0.$

3. If f(r,s,t) is differentiable and r = x-y, s = y-z, t = z-x, then show that $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=0.$

4. Let F(u,v) be any differentiable function of the two variables $u=x^2+y^2$ and $v=\sin(x^2+y^2)$. Show that w = F(u,v) satisfies $y\frac{\partial w}{\partial x}=x\frac{\partial w}{\partial y}$.

5. Consider the differentiable function f(u). If u = x-ct and c is a constant, show that $\frac{\partial f}{\partial t}+c\frac{\partial f}{\partial x}=0.$

6. The density, ρ, of a "particle" of fluid depends on time, t, and position (x,y,z). If the fluid is moving, then the particle's position depends on time (x,y,z)=(x(t),y(t),z(t)). Show that the total rate of change of density with respect to time is given by $\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+\left(\nabla\rho\right)\cdot\mathbf{v}$ where $\mathbf{v}$ is the velocity of the fluid.

7. Find $\frac{\partial z}{\partial u}$ when $u=0$ and $v=1$, if $z=\sin(xy)+x\sin(y)$ where $x=u^2+v^2$ and $y=uv$.

8. Find $\frac{\partial w}{\partial v}$ at the point $(u,v)=(-1,2)$ if $w=xy+\ln z, x=u^2/v, y=u+v$ and $z=\cos u$.

9. Consider the curve described by $f(x,y)=x^3+xy^2+\sin(xy)=5.$ Find $\frac{dy}{dx}$ using partial derivatives.

10. Consider the function f(x,y,z) where x = x(s), y = y(r,s) and z = z(r). Find an expression for $\frac{\partial f}{\partial r}$.

11. Find $\frac{dy}{dx}$ using partial derivatives if $xy+y^3-3x^2-4=0$.

12. Find $\frac{dw}{dx}$ if $w=g(u,v), u=h(x,y), v=k(x,y),$ and $y=f(x)$.

13. Find the general form of the function $w(x,y)$ whose mixed second derivative satisfies $\frac{\partial^2 w}{\partial x\partial y}=0.$

(DO NOT leave any integrals in your final answer and be sure to show all work).

14. Let $f(x,y)=\sum_{n=0}^{\infty}(xy)^n$ where $|xy|<1$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

15. Find the total differential of $f(x,y,z) = x\sin(yz) + z\exp(y^2)$.

16. Calculate the total differential, df, of the function $f(x,y,z) = x^3y^2/\sqrt{z}$.

17. If $w = w(u,v), u = u(t), v = v(t)$, then $\frac{dw}{dt} = \frac{\partial w}{\partial u}\cdot\frac{du}{dt} + \frac{\partial w}{\partial v}\cdot\frac{dv}{dt}$. Write down the similar expressions for $\frac{dw}{dt}$ in the following cases:

(a) If $w = w(u,v), u = u(x,t), v = v(t), x = x(t)$.

(b) If $w = w(u,v), u = u(x,y), v = v(x,y), x = x(t), y = y(t)$.

18. Let $x = \cos t, y = \sin t, z = \cos 2t$, and $f(x,y,z) = xy + yz + zx$. Find df/dt at $t = \pi$.

19. Find the value of $\frac{\partial x}{\partial z}$ at the point (1,-1,-3) if the equation $xz + y\ln x - x^2 + 4 = 0$ defines x as a function of the two independent variables y and z and partial derivative exists.

20. If the equation $xy + z^3x - 2yz = 0$ defines z as a function of two independent variables x, y, then use implicit differentiation to evaluate $\frac{\partial z}{\partial x}$ at $(1,1,1)$. (DO NOT try to solve for z).

21. Compute the second partial derivatives of $f(x,y) = \ln\sqrt{x^2+y^2}$ and verify that

(a) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ (b) $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

22. Some unrelated questions :

(a) Show that $z = e^{-t}\sin\left(\frac{x}{c}\right)$ satisfies the heat equation. That is $\frac{\partial z}{\partial t} = c^2\frac{\partial^2 z}{\partial x^2}$ where c is a constant and $c > 0$.

(b) Suppose that three resistors are in parallel in an electrical circuit. If the resistances are R_1, R_2, and R_3 ohms, respectively, then the net resistance in the circuit is $R = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$. Compute and interpret $\frac{\partial R}{\partial R_1}$ (do not simplify $\frac{\partial R}{\partial R_1}$).

(c) Let $f(x,y,z) = \ln(xyz^2)$. Find (i) $\frac{\partial f}{\partial x}$ (ii) f_{xzy}

23. Consider the function $p(w,x,y,z) = w^2xy - e^{wyz}$.

(a) Find the derivatives $\frac{\partial p}{\partial w}, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}, \frac{\partial^2 p}{\partial x \partial z}, \frac{\partial^4 p}{\partial w^2 \partial x \partial y}$.

(b) Which variable(s) is p most sensitive to at the point (-2,3,1,0)?

(c) Give the equation for the linearization of p at (-2,3,1,0).

24. Three unrelated chain rule problems.

(a) Verify $u(x,t) = f(x+ct) + g(x-ct)$ where c is a constant and $r = x + ct$ and $s = x - ct$ satisfies the wave equation: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

(b) Let $t = f(u,v)$ where $u = u(x,y,z,w)$ and $v = v(x,y,z,w)$. Write out the change in t with respect to y and justify whether it should be $\frac{\partial t}{\partial y}$ or $\frac{dt}{dy}$.

(c) Let $u = u(x,y)$ and $v = v(x,y)$. Express x and y in polar coordinates to show that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ (Hint : You will need to use the Cauchy-Riemann equations: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.)

25. Consider a flat square plate $\{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$. The temperature on the plate is given by $T(x,y) = x^2 y$.

(a) If an ant walks across the plate on some path $(x(t), y(t))$, what (symbolically) is $\frac{dT}{dt}$?

(b) Suppose the ant follows the parabolic path $(x(t), y(t)) = (t/2, t^2/4)$. Sketch the plate and the ant's path from $t = 0$ to $t = 2$.

(c) If the ant follows the parabolic path in (b), what is $\frac{dT}{dt}$ when the ant is at the point $(x,y) = (1/2, 1/4)$?

(d) Suppose the plate is cooling, so that $T(x,y,t) = x^2 y e^{-t}$. Now what is $\frac{dT}{dt}$ (for a general path $(x(t), y(t))$?

PART C :

12.4 Differentiability, Linearization, and Differentials,
12.7 Directional Derivatives, Gradient Vectors, and Tangent Planes

1. True or false

(a) The vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ is normal to the surface $z^2 + y^2 = \frac{x^2}{2} + yz$ at the point (x,y,z) = (2,0,1).

(b) At (x,y)=(1,0) the function $xe^{\sin(y)}$ is increasing fastest in the direction of $\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$.

(c) Let $f(x,y) = x^2 - ye^x$. The directional derivative of f at the point (0,2) in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$ is 1.

(d) A vector normal to the surface $x^2 - xy = 4 - e^x z^2$ at the point (0,3,-2) is $\mathbf{i} - 4\mathbf{k}$.

(e) The gradient ∇f to $f(x,y,z)$ points in the direction of the fastest decrease in the value of f.

(f) The function f(x,y) has extremum along the curve (x,y) = (g(t),h(t)) wherever $\frac{\partial f}{\partial x}\frac{dg}{dt} = \frac{\partial f}{\partial y}\frac{dh}{dt}$.

2. The dimensions of a box are changing in time. At a particular time $t = t_0$, all sides of the box are 5 cm long, however, the length and width of the box are increasing by 2 cm per second while the height is decreasing by 3 cm per second. At what rate is the volume changing at time $t = t_0$? Is the volume increasing or decreasing?

3. A cylinder tin can has an inside radius of 5 cm and a height of 12 cm. The thickness of the tin can is 0.2 cm.

(a) Estimate the amount of tin needed to construct the can, including the ends.

(b) Is the can volume more sensitive to variations in height or radius at the given dimensions?

4. One is trying to estimate the volume V of a circular cylinder by measuring the radius r and the height h. The relative error in the volume must be less than 0.01. Determine an upper bound on the relative errors of both the radius and the height if the relative error in the height is equal to twice the relative error in the radius.

5. The volume of a box with sides a, b and c is given by $V = abc$.

(a) Use differentials to show that $\frac{dV}{V} = \frac{da}{a} + \frac{db}{b} + \frac{dc}{c}$.

(b) If sides a and b are each decreased by 2% of their original values, **using differentials** estimate the percentage change in c such that the volume remains unchanged.

(c) Suppose $a = 1, b = 2$ and $c = 1$, estimate the percentage change in the total surface area $S = 2(ab + bc + ac)$ of the box if the sides are changed according to as in part (b).

6. Find ∇f if $f(x,y,z) = x^2 + 2y^2 + 5z^2 + z\ln x$.

7. When the function f is increasing most rapidly in the direction $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$. Give a direction in which f has zero change.

8. The derivative of f(x,y) at (1,0) in the direction of $\mathbf{i} + \mathbf{j}$ is $2\sqrt{2}$, and in the direction of $2\mathbf{i}$ it is 6. Find the direction (expressed as a vector) of greatest increase for f at (1,0).

9. Graph several level sets of $f(x,y) = x^2 + 4y^2$ including the one through (2,1). Include on this plot the vector ∇f at (2,1).

10. What is a normal vector to the surface z = f(x,y)?

11. Find the normal vector to the differentiable surface z = f(x,y).
12. Find a normal vector to the ellipsoid $x^2 + 2y^2 + 3z^2 = 9$ at (2,-1,1).

13. Let $f(x, y, z) = z^3 + x^2 z - 2xy$.
(a) Compute ∇f.
(b) If a function $z(x, y)$ is defined implicitly by $f(x, y, z) = 2,$ then evaluate z_x, z_y, z_{xy} at (1,0,1).
(c) Find the equation of the tangent plane and the normal line to the surface $f(x, y, z) = 2$ at (1,0,1).

14. Given the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$
(a) Calculate ∇f.
(b) Find the direction in which f(x,y,z) increases most rapidly and the maximum rate of change of f(x,y,z) at the point P(1,1,1).
(c) For what value of c does the level surface f(x,y,z) = c pass through the point P(1,1,1)? Find the equation of the tangent plane to the level surface f(x,y,z) = c at P(1,1,1).

15. Given $f(x, y, z) = x(1 + \cos z) - y \sin z + 2y$:
(a) Calculate ∇f.
(b) Find an equation for the tangent plane to the level surface f(x,y,z) = 2 at (2,-1,0).
(c) Estimate how much f will change if one moves a distance ds = 0.05 from the point (2,-1,0) towards the point (-1,-1,-4).

16. Let $f(x, y, z) = x^2 + xy + z^2$ and C be the ellipse $(x(t), y(t), z(t)) = (\cos t, -2\cos t, \sin t)$.
(a) Find the gradient of f.
(b) What is the direction of maximum decrease of f at the point P(1,0,1)?
(c) Find the velocity vector, $\mathbf{v}(t)$, on the curve C.
(d) Using the chain rule find $\frac{d}{dt} f(x(t), y(t), z(t))$.
(e) Let $\mathbf{u} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$. Find the equation for the set of points for which $D_{\mathbf{u}} f = 0$. Describe this set geometrically.

17. Given $f(x, y, z) = x^2 + y^2 - 3z^2 + z \ln x$.
(a) Is there a direction $\mathbf{A}$ in which the rate of change of f equals 10 at the point $P_0(1,1,1)$?
(b) Find the equations for the tangent plane and the normal line at the point P_0 to the surface $f(x, y, z) = 0$.
(c) Approximate the change in f if one moves from P_0 a distance 0.1 toward the point (-1,0,3)?

18. Let $g(x, y) = \cos(\pi xy) + xy^2$.
(a) In what direction(s) is the derivative of g at (1,1) equal to 0?
(b) Find the direction(s) in which the function g increases most rapidly at (1,1).
(c) At the point (1,1), is g more sensitive to changes in x or to changes in y? Explain.

(d) Find the line normal to the surface z = g(x,y) at the point (1,1,0).

19. Consider the function $f(x,y) = \exp(-2x-3y)$.

(a) Compute the linearization of f(x,y) at the origin.

(b) If we use this linearization to approximate the function in the square region defined by $-1 \le x \le 1$ and $-1 \le y \le 1$, give an approximation to the error.

(c) Now consider the point (1,1). At this location, in what direction does $f(x,y)$ change the fastest?

(d) What is that rate of change?

(e) If one moves 0.1 distance units from the point (1,1) in a direction defined by the unit vector $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$, by approximately how much does f(x,y) change?

20. Suppose the elevation on a hill is given by $f(x,y) = 100 - y^2 - 4x^2$.

(a) Sketch what a road would look like (when viewed from above) if it was to be built on the hill at a constant elevation of 92 feet.

(b) In which (horizontal) direction will the rain run off at the point (1,2)?

(c) Sketch the direction you found in (b), on your sketch in (a), and explain how it relates to the shape of the road. (Note that (1,2) is at an elevation of 92 feet and is, therefore, on the road.)

21. Consider the paraboloid $f(x,y) = x^2 + y^2$. For the following, give your answers in terms of a and b.

(a) Find the equation of the plane tangent to f(x,y) at the point (a,b,c).

(b) Find the equation for the line where the tangent plane in part (a) intersects the x-y plane.

22. Let $f(x,y) = 2x^3 - 3x^2 + y^2 - 2y + 3$.

(a) Calculate ∇f.

(b) Find the direction in which f is decreasing most rapidly at (2,3).

(c) Find all critical points of f and classify each as either a maximum, a minimum, or a saddle point.

23. A probe is moving through space, being bombarded with cosmic radiation. The strength of the radiation bombardment is given by a function R(x,y,z) and the probe is moving along the path $\mathbf{r}(t)$. At a given time t^* you know that $\mathbf{r}(t^*) = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $\mathbf{v}(t^*) = \mathbf{j} - 3\mathbf{k}$ and $\mathbf{a}(t^*) = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$; you also know that R(1,-2,-1) = 5 and $\nabla R\big|_{(1,-2,-1)} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.

(a) What is the rate of change of the radiation with respect to <u>time</u> that the probe experiences?

(b) What is the rate of change of the radiation with respect to <u>distance</u> that the probe experiences?

(c) Suppose the probe is receiving an overdose of radiation, so mission control wants to send a command to change direction. Which direction should they redirect the probe to so that the radiation dosage decreases most rapidly? (You may assume instantaneous communication and redirection.)

(d) Suppose(ignore part (c)) that mission control wants to keep the probe at the current level of radiation. Unfortunately, due to budget cuts, the probe was built by students from CSU, rather than CU; consequently, the thrusters that maneuver the probe in the $\mathbf{k}$ direction have broken. Can they still move the probe in a direction that keeps R constant? If not, why not? If so, find the direction.

24. A remote-control submarine is moving through the ocean along a path $\mathbf{r}(t)$; the temperature of the water is given by a function T(x,y,z). At a given time t^* you know that $\mathbf{r}(t^*) = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $\mathbf{v}(t^*) = \mathbf{j} - 3\mathbf{k}$ and $\mathbf{a}(t^*) = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$; you also know that T(1,-2,-1) = 5 and $\nabla T\big|_{(1,-2,-1)} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.

(a) Calculate the rate of change of the temperature with respect to **time** that the submarine experiences (at time t^*), or explain why it cannot be calculated.

(b) Calculate the rate of change of the temperature with respect to **distance** that the submarine experiences (at time t^*), or explain why it cannot be calculated.

(c) Suppose T = 5 is too cold for the submarine to function properly, so its controllers want to send a command to change direction. Which direction should they redirect the submarine to so that the temperature increases most rapidly? (You may assume instantaneous communication and redirection.)

(d) Suppose (ignore part (c)) that T = 5 is a perfect operating temperature for the submarine, so the controllers want to keep it at that temperature. However, seaweed has jammed some of the control surfaces, so the submarine cannot be maneuvered in the $\mathbf{i}$ direction. Can the controllers still move the submarine in a direction that keeps T constant? If so, what direction? If not, why not?

25. The temperature distribution of the antarctic ice is given by a function T(x,y). A peguin walks along a 2-D path $\mathbf{r}(t)$. At time t = 2.3, the penguin is at the point (-1,4) and has velocity $\mathbf{v}(2.3) = \mathbf{i} - 2\mathbf{j}$ km/hr. You also know that T(-1,4) $= -40^\circ$ and $\nabla T\big|_{(-1,4)} = 3\mathbf{i} + 2\mathbf{j}$.

(a) Calculate the rate of change of temperature with respect to time, in $^\circ/\mathrm{hr}$, that the penguin is experiencing at t = 2.3, or explain why it cannot be calculated from the information given.

(b) Calculate the rate of change of temperature with respect to distance, in $^\circ/\mathrm{km}$,that the penguin is experiencing at t = 2.3, or explain why it cannot be calculated from the information given.

(c) The penguin has just realized that it's very cold in the Antarctic and so would like to change direction and go somewhere warmer. What direction should the penguin go to warm up the fastest?

(d) On further reflection, the penguin has decided that -40° is very good temperature (it's a very well-educated penguin and knows that -40 is where the Fahrenheit and Celsius scales align). What direction should the penguin go to keep the temperature constant?

26. An open cockpit plane is flying in a circular path about the airport while waiting for clearance to land. Measured from airport, its path is described by $\mathbf{r}(t) = 10\cos(2t)\mathbf{i} + 10\sin(2t)\mathbf{j} + 10\mathbf{k}$ where distances are measured in kilometers and time is measured in minutes. The temperature in degree C, as a function of position, is given by T(x,y,z) = 0.3xy+0.5z. Answer the following questions assuming the plane is located at (0,10,10).

(a) What rate of temperature change does the pilot experience in $^\circ\mathrm{C}/\mathrm{min}$?

(b) In $^\circ\mathrm{C}/\mathrm{km}$?

(c) If the pilot suddenly turned and flew toward the airport while maintaining its altitude, by approximately how many degrees C would the temperature change after the pilot travels 0.1 km in the new direction?

27. Suppose the Celsius temperature distribution in the x-y plane is given by $T(x,y) = xe^{2y}$.

(a) If you were standing at the point (2,0), in which direction (unit vector) would you move to warm up as quickly as possible? Please be explicit.

(b) If you were standing at the point (2,0), give the unit vector(s) associated with the direction in which the temperature would remain constant.

(c) Find the linear approximation L(x,y) to T(x,y) at the point (2,0).

(d) Estimate the error of the approximation $L(x,y) \approx T(x,y)$ for the rectangle $R = \{1.9 \le x \le 2.1, -0.1 \le y \le 0.1\}$. (You do not have to simplify your error estimate.)

(e) Without doing any additional calculations, find the equation of the tangent plane to $z = xe^{2y}$ at the point (2,0,2).

28. On a particular day, the temperature in Colorado is given by $T(x,y) = xy^2$, where the origin is located at the Starbuck's along Arapahoe Ave. near 28^{th} Street. Distances are measured in miles.

(a) If I sit at the location (1,1), give the unit vector associated with the direction I should go to warm up as quickly as possible.

(b) At the same location, give the unit vector associated with the direction I can go to stay at the same temperature.

(c) If I move from my current location of (1,1) to a new location (1.1,1.1), by approximately how much (this means use Calculus!) has the temperature changed?

29. Water is an incompressible fluid for which the pressure P, velocity v, density ρ, and height of the fluid z above some reference elevation are related by Bernoulli's equation

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{constant}$$

Where g is the constant acceleration due to gravity. Assume that, under some circumstance, the density of the water is 1000 kg/m^3, the velocity is 200 m/s, and the height is 1 m. Approximately what is the change in pressure of the water if the density is increased by 10 kg/m^3, the velocity is decreased by 1 m/s, and the elevation is increased by 0.1 m? Assume that g = 10 m/s^2.

30. The gravitational potential energy of a body of mass m in the presence of mass M is given by $P(x,y,z) = \frac{-GMm}{\sqrt{x^2+y^2+z^2}}$, where the origin is at the center of mass M (and G is the universal gravitational constant).

(a) Describe the level sets of P geometrically. Be sure to explain how the level sets change as the value of P is changed. Draw a sketch to illustrate your answer.

(b) Find a unit normal vector to the level set where $P(x,y,z) = P_0$ (for a general (x,y,z) point on this set).

(c) The mass m is in "free-fall" if it follows a path that decreases P the most rapidly. Using principles of Calc III and your answers to (a) and (b), explain what direction the mass m will go when in free-all (and why). Describe this direction geometrically and how it relates to the level sets. Explain briefly why this makes sense physically.

31. Astronomers have discovered a new planet in orbit around a distant star. From their observations, they have estimated that the planet's orbit period is $T = 4 \pm 0.05$ years, and its semi-major axis is $a = 2 \pm 0.05\,AU$. (The AU is an astronomical unit of the length $\approx 1.5 \times 10^{11}$ m.) According to Kepler's third

law, $\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$, where G is the universal gravitational constant and M is the mass of the star-planet system. Using this, and the values T= 4, a = 2, the mass was determined to be 10^{30} kg.

(a) Estimate the error in the calculated mass; the value of $\frac{4\pi^2}{G}$ is $2\times10^{30}\ \frac{\text{kg}\cdot\text{year}^2}{\text{AU}^3}$.

(b) With the vast quantities of grant money now coming to them, the astronomers have hired a student to analyze the data and improve the error estimates on a and/or T. With a conference deadline approaching, the student has time to work on only one; which measurement, a or T, will give the astronomers the better return, in terms of improved accuracy of M?

32. A hiker is on a mountain given by the function $z = f(x,y) = 106 - x^2 - y^2$ and is standing at the point (3,4).

(a) A thunderstorm is beginning to form, and knowing that lightning is the second leading cause of weather-related death in Colorado, the hiker wants to get down from the mountain as quickly as possible. In which direction should she hike to descend the mountain the fastest?

(b) Find the first order Taylor series approximation for f(x,y) at the point (3,4).

PART D:
12.8 Extreme Values and Saddle Points

1. True or false

(a) If (x^*, y^*) is a critical point of f and $f_{yy}\big|_{(x^*,y^*)} < 0$ and $f_{xy}\big|_{(x^*,y^*)} < 0$, then (x^*, y^*) is a maximum of f.

(b) If there is a direction $\mathbf{u}$ such that $D_{\mathbf{u}}f\big|_{(x^*,y^*)} = 0$, then (x^*, y^*) is a max or min of f.

(c) If $f_{xy}(a,b) < 0$ and $f_{yy}(a,b) < 0$ then $f(x,y)$ has either a maximum or a saddle at (a,b).

(d) Suppose f(x,y) has continuous second derivatives at (0,0) and $f_x(0,0) = 0, f_y(0,0) = 0, f_{xx}(0,0) = 2,$ $f_{yy}(0,0) = -2,$ then at the point (0,0), the function f has a saddle point.

(e) The only critical points of $f(x,y) = x^3 + y^3 - 3x - 3y$ are (-1,-1) and (1,1).

(f) Suppose f(x,y) is a smooth function on a neighborhood R of (0,0). Furthermore, suppose that $f(0,0) = 0, f_x(0,0) = 0, f_y(0,0) = 0,$ and the product $f_{xx}(0,0)f_{yy}(0,0) = -5.$ Then there exist points (x_1, y_1) and (x_2, y_2) in R such that $f(x_1, y_1) < 0$ and $f(x_2, y_2) > 0.$

(g) If a function f(x,y) is twice differentiable in a region surrounding point P(a,b) and has a local maximum at (a,b), then $f_{yy}\big|_P < 0.$

(h) If a function f(x,y,z) has a critical point at P(a,b,c), then $\nabla f\big|_P = 0.$

(i) If $f_{xy}(a,b) < 0$ and $f_{xx}(a,b) > 0$ then $f(x,y)$ has a minimum at (a,b).

(j) Suppose that (3,2) is a critical point of the function $f(x,y)$, and that all the partial derivatives of f exist and are continuous. If $f_{xx}(3,2) = -2$ and $f_{yy}(3,2) = 3,$ then, at the point (3,2), the function f(x,y) has a saddle point.

(k) A critical point to f(x,y) (meaning that it satisfies $f_x = f_y = 0$) is either a maximum, a minimum, or a saddle point.

(l) If a function f(x,y) at a point (x,y) features $f_x = f_y = 0, f_{xx} < 0, f_{yy} < 0,$ then f has a local maximum at this location.

(m) The function $f(x, y) = x^2 + ye^x$ has no critical points.

2. How many critical points does the function $f(x, y) = x^3 - 3x + 2y + 1$ have?

3. Find all local maxima, local minima, and saddle points of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8.$

4. Find all the local maximum, minimum and saddle points of $f(x, y) = x^3 - 3xy + y^2 + y + 1.$

5. Locate and classify all critical points of $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy.$

6. Locate and classify all critical points of the function $f(x, y) = x^3 + 3xy^2 + 9y^2 - 75x + 2.$

7. Let $f(x, y) = y^2 + 2xy + 3x^2 - x^4.$

(a) Find all critical points of f.

(b) Classify the critical points from (a) as maxima, minima or saddle points.

(c) Write down the complete set of equations that you would use in the method of Lagrange Multipliers to find the critical points of f on the circle $x^2 + y^2 = 4.$ Identify all of the unknowns in these equations, but you do **not** need to solve them.

8. Let $f(x, y) = xy - 2x^4 - y^2.$

(a) Find the direction in which f is increasing most rapidly at (1,8).

(b) Find all the critical points of f(x,y).

(c) Classify each critical point as either a maximum, minimum or saddle point.

9. Consider the function $f(x, y) = x^3 + x^2y + y^2.$

(a) Find all local maxima, minima, and saddle points (if any) for f(x,y).

(b) Find the linear approximation to f(x,y) at (1,1).

(c) For what directions $\mathbf{u}$ is the directional derivative $D_{\mathbf{u}}f\,(=\frac{df}{ds})$ of f equal to zero at the point (1,1)?

10. Let $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy.$

(a) Find the direction in which f is increasing most rapidly at the point (1,-3/2).

(b) Find all local maxima, local minima, and saddle points of f(x,y).

11. Let $f(x, y) = 2x^2 + x - y + 2y^2.$

(a) Find all maxima, minima and saddle points for f(x,y).

(b) Find the absolute maximum and minimum of f(x,y) on the curve $x^2 + y^2 = 8.$

(c) Use your results from parts (a) and (b) to give the absolute maximum and the absolute minimum for f(x,y) on the closed region $x^2+y^2 \le 8$.

12. Consider the function $f(x,y)=e^{x^2+2y^2}$.

(a) Find and classify all critical points of f(x,y).

(b) Now, find the global min/max values of f(x,y) over the region $R=\{(x,y):2x^2+2xy+5y^2 \le 9\}$.

13. Find all the local maxima, local minima, and saddle points of the functions in (a), (b), and (c).

(a) $f(x,y)=\frac{1}{x}+xy+\frac{1}{y}$ (b) $f(x,y)=y\sin x$ (c) $f(x,y)=e^{2x}\cos y$

14. Consider the function $f(x,y)=x^2+y^2-2\cos(y)$.

(a) Find all local maxima, minima and saddle points (and state clearly which type they are).

(b) Are any of the maxima or minima global max/mins? [Hint: look at the values of f as x and y $\to \pm\infty$.]

(c) Sketch the domain $D=\{(x,y):x^2+(y-1)^2 \le 4\}$.

(d) Find the global max and min of f over the domain D.

15. Let $f(x,y)=x^3-y^3+3xy$.

(a) Find all local minima, maxima and saddle points of f(x,y).

(b) Let $g(x,y)=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}$. Find the location of the optimal values(s) of $g(x,y)$ on the line $y=2x+a$. (Report your answer in terms of a.)

(c) Find the linear approximation of f(x,y) at the point (2,1).

(d) Find an error bound of the linear approximation in (c) on the rectangle $1.5 \le x \le 2.5$, and $0.75 \le y \le 1.25$.

(e) Write down the cubic approximation of f(x,y) at (0,0).

16. The porosity of the soil in a circular field, $x^2+y^2 \le 1$, is given by $p(x,y)=4x^2+2y^2-2y$. Find the locations in the field where water will drain the fastest and slowest(i.e. the absolute maximum and minimum porosity).

17. Consider the function $f(x,y)=4xy$ on the closed region $R=\{(x,y):x^2+y^2 \le 8\}$

(a) Sketch R , label the intercepts.

(b) Find all absolute extrema of f(x,y) on R.

(c) On your sketch of R, show the locations of the absolute extrema and clearly label them as maximum or minimum.

18. You have just field a mining claim on some property near Gold Hill. The boundaries of your claim are described by the lines x = 0, y = 3 and y = x in the first quadrant. You don't know it, but the density of

gold (in micro grams per cubic centimeter) on your claim is described by $f(x,y) = x^2 - 4x + y^2 - 2y + 5$. At what locations on your claim would your mining efforts be the most, and least, productive?

19. Loedz is a very creative spider. He built an elaborate web one night in an inspired frenzy which can be modeled as the function $x^2 + 3y - y^3$. His less creative(more practical) friend spider Jonesy criticized his design saying that leaves would get caught in the troughs of thing. Loedz is also a smart spider taking Calc 3. Help Loedz defend his design by computing the following.

(a) Identify all critical points of Loedz web.

(b) For each critical point in a, identify each as a maximum, a minimum, a saddle point, or inconclusive, showing all work.

20. A flat plate in the first quadrant is bounded by the triangular region R given by the lines x = 0, y =0 and y = 2-x. The plate is heated so that the temperature at the point (x,y) is given by $f(x,y) = x^2 + y^2 - x - 2y$.

(a) Find all local maxima and minima of the function $f(x,y) = x^2 + y^2 - x - 2y$.

(b) Find all of the (x,y) points on the boundary of R where f could have local maxima and minima.

(c) Make a table of all of the points (x,y) you found in part (a) and (b) and the value of the function f(x,y). What are the (x,y) coordinates, and the temperature, of the hottest and coldest points on the plate?

21. What values of a and b minimize the value of $\int_a^b (x^2 - x - 1)dx$?

22. Determine the closed 3-dimensional domain D for which the following integral $\iiint_D (1 - x^2 - 2y^2 - 3z^2)dxdydz$ has the largest value.

PART E:
12.9 Lagrange Multipliers

1. Find the point on the surface $x^2 + 4y + 4z = 8$ closest to the origin.(Hint : consider the square of the distance).

2. Find the point(s) on the surface $z^2 - xy = 4$ closest to the origin. Hint: consider the square of the distance.

3. Find the points on the surface $z^2 = xy + 4$ closest to the origin. (Hint : consider the square of the distance.)

4. Find the points on the curve $x^2 + xy + y^2 = 1$ in the xy-plane that are nearest to and farthest from the origin.

5. By means of Lagrange's multipliers (i.e. minimizing the function $f(x, y, u, v) = (x-u)^2 + (y-v)^2$ subject to the constraints $y = x + 1$ and $u = v^2$), find the minimum distance in the x,y-plane from the line $y = x + 1$ to the parabola $x = y^2$.

6. Find the point on the graph of $z = x^2 + y^2 + 10$ nearest the plane $x + 2y - z = 0$. (Hint: There is no need for Lagrange multipliers in this problem.)

7. Find the dimensions of the rectangular box of greatest volume that can be inscribed in a sphere of radius $\sqrt{3}$.

8. The temperature of a fluid at the point (x,y,z) is given by $T(x, y, z) = x^2 + yz + 1$. Find the coldest temperature obtained at points in the fluid that are exactly one unit away from the origin.

9. The base of an open-top rectangular box costs \$3 per square meter to construct, while the sides cost only \$1 per square meter. Find the dimensions of the box of greatest volume that can be constructed for exactly \$36.

10. A large closed rectangular box sits in the first octant with three faces in the coordinate planes. The vertex of this box that does not touch any of the coordinate planes(the one out in space) intersects the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where a,b and c > 0. Find the position of the vertex that maximizes the volume of the box.

11. You are asked to construct a grain silo of volume 1000 m^3 from a cylinder of radius r and height h, topped by a hemisphere of radius r. The cost to build the silo is \$1/m^2 for the sidewalls, and \$3/m^2 for the hemispherical top. (Hint : the volume of a sphere is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.)

(a) What is the function describing the total volume of the silo, V, in terms of r and h?
(b) what is the function describing the total cost of the silo, C, in terms of r and h?
(c) Your task is to build the silo for minimum cost. Set up the problem using Lagrange multipliers.
(d) Using your result from part (c) , find the values of r and h that give the silo of minimum cost.

12. A mother puts her child on an amusement park ride that takes the child along a path described by the equation $x^2 - 2x + y^2 - 4y = 0$. While the child is on the ride, the mother stands at the location (0,0). Find the minimum and maximum distances from the mother to the child. What are the coordinates of the child at the minimum and maximum distances.

13. A submarine plunges into the depths of the ocean with increasing pressure on its hull. When the submarine comes to the rest, the cross-section of its center can be described by the ellipse $4x^2 + y^2 = 16$ and the pressure by the function $P(x, y) = 4x^2 + 2y^2 + 1$. Locate the points on this cross-section that experience the greatest and lowest pressure.

14. The temperature on the surface of an ellipsoidal object $4x^2 + y^2 + 4z^2 = 16$ is given by $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the maximum and minimum temperatures on the surface of the object. [It may help to know that $\sqrt{3} < 16/9$.]

15. In quantum mechanical system consisting of an electron contained within a right-circular cylinder, the minimum kinetic energy that the electron can have is given by

$$E = \frac{\hbar}{2m}\left(\frac{k}{R^2} + \frac{\pi^2}{H^2}\right) = C\left(\frac{k}{R^2} + \frac{\pi^2}{H^2}\right)$$

where R and H are the radius and height, respectively, of the cylinder, and $\hbar$, k and m (and, therefore, C) are all constants. Find the ratio R/H that gives the minimum energy E for a cylinder of fixed volume V. [Hint: start as if you were going to find R and H explicitly in terms of V, but as you proceed remember that we only need R/H at the end.]

16. The cost of designing the bottom of a rectangular swimming pool is 5 times as much (per square meter) as its four sides. If the pool is to hold 20 cubic meters of water, find the optimum dimensions of the pool that will minimize the total cost of design.

PART F :

12.10 Taylor's Formula

1. Compute the quadratic approximation of $f(x, y) = \frac{1}{1 - x - y}$ about (0,0).

2. Calculate the quadratic approximation for $f(x, y) = \frac{1}{1 + x - y}$ about $(0, 0)$.

3. Compute the quadratic Taylor expansion of $f(x, y) = e^{2x}\cos(2y)$ about $(0, 0)$.

4. Let $f(x, y) = \ln(x - 2y + 1)$.

(a) Compute the quadratic Taylor expansion of f(x,y) about (0,0).

(b) Suppose that x and y are each functions of t, and that x(4) = y(4) = 0. Suppose also that dx/dt = -3 and dy/dt = 5 when t = 4. Use the chain rule to calculate df/dt when t = 4.

5. Let $f(x, y) = e^{-(x^2 + 2y)}$.

(a) Find the quadratic Taylor expansion of f about the point (0,0).

(b) Find the equation for the tangent plane at (0,0,1) on the surface of z = f(x,y).

6. Consider the function $f(x, y) = (x^2 + y^2 - 3)e^x$.

(a) Find all critical points of f. For each critical point, determine whether it is a maximum, a minimum, or a saddle point.

(b) Find the quadratic approximation of f near (0,0).

7. Let $f(x, y) = e^x \sin y$.

(a) Using Taylor's formula, find the cubic approximation to f(x,y) at the origin.

(b) What is the equation for the plane tangent to the surface z = f(x,y) at the origin?

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