INSTRUCTIONS: Electronic devices are not permitted during the exam. Write your name, your instructor's name, and your recitation number on the front of your bluebook. Start each problem on a new right-hand page. Justify your work clearly and $\boxed{\text{box}}$ your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) For each of the following unrelated questions, answer either ALWAYS TRUE or NOT ALWAYS TRUE. No justification is necessary.
   (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.
   (b) If $\mathbf{A} \times \mathbf{B} = \mathbf{C} \times \mathbf{B}$ and $\mathbf{B} \neq \mathbf{0}$, then $\mathbf{A} = \mathbf{C}$.
   (c) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = (\mathbf{B} \times \mathbf{A}) \cdot \mathbf{B}$.
   (d) If an object moves on the surface of a sphere, then its acceleration vector, $\mathbf{a}$, is orthogonal to its velocity vector $\mathbf{v}$.

2. (20 points) Consider two glass plates (planes). The first plane, P_1, intersects the principle axes at the locations $(1,0,0)$, $(0,1,0)$, and $(0,0,2)$. The second plane, P_2, is parallel to the x-axis and intersects the remaining two principle axis at the same points as plane P_1.
   (a) Determine the standard equation of plane P_1 and its unit normal vector $\mathbf{n}_1$.
   (b) Determine the standard equation of plane P_2 and its unit normal vector $\mathbf{n}_2$.
   (c) Determine the cosine of the angle between the plates, $\cos\theta$.
   (d) A laser beam at the origin is aimed perpendicular to plane P_1 and pierces plane P_2 at point B. What are the coordinates of point B.

3. (20 points) Consider a particle moving at a constant speed along a curve in space described by $\mathbf{r}(t)$. Although you do not know the form of the function $\mathbf{r}(t)$, you do know that at a given time t^*, $\mathbf{r}(t^*) = 0\,\mathbf{i} + 3\,\mathbf{j} + 4\,\mathbf{k}$, $\mathbf{v}(t^*) = 5\,\mathbf{i} + 5\,\mathbf{j} + 0\,\mathbf{k}$, and $\mathbf{a}(t^*) = 0\,\mathbf{i} + 0\,\mathbf{j} + 7\,\mathbf{k}$. If you can, calculate the following quantities at time $t = t^*$. Otherwise, clearly state that there is insufficient information to perform the calculation.
   (a) The unit tangent $\mathbf{T}$.
   (b) The unit normal $\mathbf{N}$.
   (c) The curvature κ.
   (d) The torsion τ.
   (e) Now, assume the time is $t = 2t^*$. What is $\mathbf{v} \cdot \mathbf{a}$?

4. (20 points) Annie the ant likes extreme sports. One day she sees a bicycle wheel approaching and she jumps on as it passes by. She rides on the wheel for two full revolutions, then jumps off when she is back at ground level. The path Annie takes as she rides on the wheel is described by $\mathbf{r}(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j}$ where $\mathbf{i}$ is parallel to the ground surface and $\mathbf{j}$ is perpendicular to the ground surface.
   (a) How long does Annie's ride last?
   (b) How far does Annie travel while on the wheel?

5. (20 points) Consider the quadric surfaces defined by $Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$, where the coefficients A through G only have the values -1, 0, or 1. For each of the surfaces described below, you need to assign values to the coefficients and then write out the simplest equation for the quadric surfaces.

   (a) Paraboloid centered on the negative x-axis.
   (b) A set of cones centered on the y-axis.
   (c) Hyperbolid of two sheets centered on the x-axis.
   (d) Hyperboloid of one sheet centered on the y-axis.

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[\,1+(f'(x))^2\,]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$