Spring 09 Exam 2

1a) height $f(x,y) = xy - \frac{x^3}{3} - \frac{y^3}{3}$

$\nabla f = \langle y - x^2, x - y^2 \rangle = 0 \Rightarrow y = x^2$ and $x = y^2$

so $x, y \geq 0$ $\quad y = y^4 \Rightarrow y = 1$ or 0. then $x = 1$ or 0

$\boxed{\text{points: } (0,0), (1,1)}$

1b) $f_{xx} = -2x$, $f_{yy} = -2y$ and $f_{xy} = 1 \Rightarrow f_{xx} f_{yy} - f_{xy}^2$

$4xy - 1 \Big|_{(0,0)} = -1 < 0 \Rightarrow$ (saddle)

$4xy - 1 \Big|_{(1,1)} = 3 > 0$

$f_{xx} \Big|_{(1,1)} = -2 < 0$ $\Rightarrow$ (max).

point	f value	classification
(0,0)	0	saddle
(1,1)	1/3	loc max (hilltop)

1c) (0,0) to (1,1). so travel along line $y = x$. (direction $\frac{i}{\sqrt{2}} + \frac{j}{\sqrt{2}}$)

max slope on this line...

$D_u f = \langle y - x^2, x - y^2 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \frac{1}{\sqrt{2}}(y - x^2 + x - y^2)$

and we are on the line $y = x$. so $\text{slope}(x) = \frac{1}{\sqrt{2}}(2x - 2x^2)$

max slope when $\text{slope}'(x) = 0$

$\frac{1}{\sqrt{2}}(2 - 4x) = 0 \Rightarrow 4x = 2 \Rightarrow x = \frac{1}{2}$

then $\text{slope}(\frac{1}{2}) = \frac{1}{\sqrt{2}}(1 - 2\frac{1}{4}) = \boxed{\frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}}$

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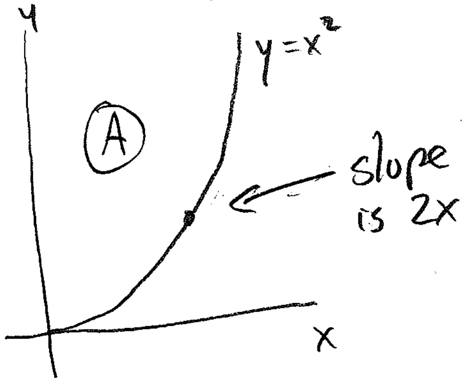
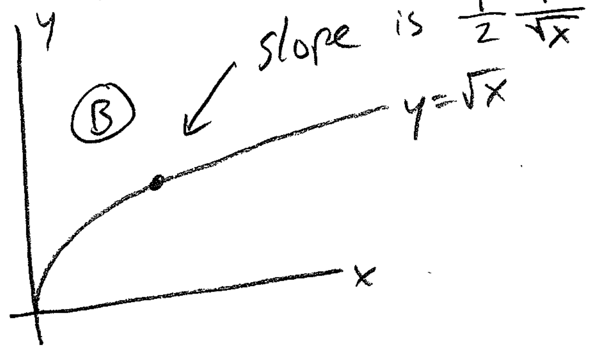
2a) $T(x,y) = 50 + xy + y^2$
$\nabla T = \langle y, x+2y \rangle$

A goes on $y=x^2$
B goes on $y=\sqrt{x}$
$|v| = 10$





meet at (1,1), slope vector

$A_{slope}: \langle 1,2 \rangle$

$B_{slope}: \langle 2,1 \rangle$

normalized: $\frac{1}{\sqrt{5}}\langle 1,2 \rangle$

normalized: $\frac{1}{\sqrt{5}}\langle 2,1 \rangle$

$D_{T_A}\nabla T = \langle 1,3 \rangle \cdot \frac{1}{\sqrt{5}}\langle 1,2 \rangle$
$= \frac{1}{\sqrt{5}}(1+6) = \boxed{\frac{7}{\sqrt{5}} \ (A)}$

$D_{T_B}\nabla T = \langle 1,3 \rangle \cdot \frac{1}{\sqrt{5}}\langle 2,1 \rangle$
$= \frac{1}{\sqrt{5}}(2+3) = \frac{5}{\sqrt{5}} = \boxed{\sqrt{5} \ (B)}$

2b) $\frac{dT}{dt} \approx \frac{\Delta T}{\Delta t} \Rightarrow \Delta T \approx \frac{dT}{dt}\Delta t = \frac{dT}{ds}\frac{ds}{dt}\Delta t = \frac{dT}{ds}|v|\Delta t$
$= \frac{7}{\sqrt{5}}(10)(0.1) = \boxed{\frac{7}{\sqrt{5}}}$

2c) $\frac{dT}{ds} \approx \frac{\Delta T}{\Delta s} \Rightarrow \Delta T \approx \frac{dT}{ds}\Delta s = \sqrt{5}(0.1) = \boxed{\frac{\sqrt{5}}{10} \text{ or } \frac{1}{2\sqrt{5}}}$

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3.) $f(x,y) = 9 - \frac{x^3}{3} - \frac{y^3}{3}$ $\quad x \geq 0,\ y \geq 0$ $\quad$ hway: $xy = 4$

$g(x,y) = xy - 4$

$\nabla f = \lambda \nabla g \Rightarrow$

$$\left.\begin{aligned} x&: -x^2 = \lambda y \\ y&: -y^2 = \lambda x \\ \lambda&: xy = 4 \end{aligned}\right\} \text{3eq, 3 unk...}$$

so $-x^2 = \lambda y \Rightarrow \lambda = \frac{-x^2}{y}$ and $-y^2 = \lambda x \Rightarrow \lambda = \frac{-y^2}{x}$

$\lambda = \lambda$ so $\frac{-x^2}{y} = \frac{-y^2}{x}$ ($\lambda = 0$ not valid, because $x = y = 0$ doesn't satisfy $xy = 4$.)

$\Rightarrow x^3 = y^3$

$x = y$ into $xy = 4$

$x^2 = 4$

$x = \pm 2$ discard -2 since $x \geq 0$.

$\boxed{\text{then } (x,y)_{\text{max}} = (2,2)}$

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4) $f(x,y,z) = \dfrac{x^2y^3}{z}$ each x, y, z 10% high!

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = 0.1$$

$$\frac{df}{f} = \frac{f_x\,dx + f_y\,dy + f_z\,dz}{f} = \frac{f_x}{f}dx + \frac{f_y}{f}dy + \frac{f_z}{z}dz$$

$$f_x = \frac{2xy^3}{z} \quad f_y = \frac{3x^2y^2}{z} \quad f_z = \frac{-x^2y^3}{z^2}$$

$$\frac{df}{f} = \frac{\frac{2xy^3}{z}}{\frac{x^2y^3}{z}}dx + \frac{\frac{3x^2y^2}{z}}{\frac{x^2y^3}{z}}dy + \frac{\frac{-x^2y^3}{z^2}}{\frac{x^2y^3}{z}}dz$$

$$= 2\frac{dx}{x} + 3\frac{dy}{y} - \frac{dz}{z}$$

$$= 2(0.1) + 3(0.1) - (0.1) \boxed{= 0.4}$$

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5a) $f(x,y) = x^3 + y^3 + x^2y^2$

1^{st} order near $(1,1)$

$f_x = 3x^2 + 2xy^2$

$f_y = 3y^2 + 2yx^2$

$f(1,1) = 3$, $f_x(1,1) = 5$, $f_y(1,1) = 5$

$$f(x,y) \approx 3 + \left[(x-1)(5) + (y-1)(5)\right]$$

$$= 3 + 5x - 5 + 5y - 5$$

$$= \boxed{-7 + 5x + 5y \approx f(x,y)}$$

5b) $f(1.1, 1.1) = -7 + 5.5 + 5.5 = 11 - 7 \boxed{= 4}$

5c) $|x-1| \leq 0.1$

$|y-1| \leq 0.1$

$$|E(x,y)| \leq \frac{1}{6} M\left(|x-x_0| + |y-y_0|\right)^3$$

$f_{xx} = 6x + 2y^2$

$f_{xxx} = 6$

$f_{xxy} = 4y$

$f_{yy} = 6y + 2x^2$

$f_{yyx} = 4x$

$f_{yyy} = 6$

$M = \max\{|f_{xxx}|, |f_{xxy}|, |f_{xyy}|, |f_{yyy}|\}$

$M = \max\{|6|, |4y|, |4x|, |6|\}$

$= 6$

$$|E(x,y)| \leq \frac{1}{6} 6\left(0.1 + 0.1\right)^3 = (0.2)^3 \Rightarrow \boxed{|E(x,y)| \leq 0.008}$$