Calc. 3
Exam 3 review

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Exam 3 is on Wednesday, 11/28, so now is a good time to begin studying if you haven't already done so. To help in your studying I've created the following review sheet with a list of important topics and formulas to get you started. As always, I believe one of the best ways to study is to do some of the old exams which are on the website. The exam will cover Sections 13.1-14.2 (excluding 13.5) from the textbook.

Capter 13

Chapter 13 covers the topic of multiple integrals in various coordinate systems including cartesian, cylindrical and spherical coordinates as well as general coordinate transformations and the use of the Jacobian operator to account for the stretching which takes place in coordinate transformations.

13.1: Double integrals (p. 1001)

Key topics:

1. Double integrals over rectangles
2. Double integrals as volumes
3. Double integrals over non-rectangular regions
   - Determining best order of intergration
   - Determining limits

13.2: Areas, Moments, and Centers of Mass (p. 1012)

Area of a closed, bounded region R:

$$A = \iint_R dA$$

Average value of f over R:

$$\text{Ave.} = \frac{1}{\text{area of } R} \iint_R f \; dA$$

Mass, first moments (used for centers of mass), center of mass, second moments (moments of inertia) and radii of gyration are included in table 13.1 (p. 1014) and you should be familiar with them all.

Centroids, found the same way as center of mass, but setting $\delta(x, y) = 0$.

13.3: Double Integrals in Polar Form (p. 1020)

Key topics:

1. Polar coordinates ($x = r\cos(\theta),\ y = r\sin(\theta),\ r = x^2 + y^2,\ \theta = \arctan(y/x)$)
2. $dA = r \; dr \; d\theta$
3. Determining limits of integration in polar coordinates
4. Changing cartesian integrals to polar coordinates (Draw a pictureto help with the limits!)

13.4: Triple integrals in Rectangular Coordinates (p. 1026)

This is mostly the same as double integrals except finding the limits of integration can be more difficult. It helps to think of an integration in a certain variable as "smashing" the region of integration in that direction. For example, if we integrate in z over the cube $0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 1$ we have effective "smashed" the cube in the z-direction, leaving the square $0 \leq x \leq 1,\ 0 \leq y \leq 1$ for our last 2 integrations. Also note that a triple integral can be used to find the volume of a region and that our average value equation translates to triple integrals just as you would expect.

13.6: Triple Integrals in Cylindrical and Spherical Coordinates (p. 1039)

Be familiar with the coordinate transformations between cartesian, cylindrical, and spherical coordinates (table on p. 1044). It would be a good idea to get some practice finding limits for spherical integrations since these are sometime difficult to determine. Exercises 33-38 on page 1046 offer some practice with this. Finally, be sure not to forget to change your incremental volumes (dV) appropriately:

$$dV = dx \; dy \; dz = r \; dr \; d\theta \; dz = \rho^2 \sin(\phi) \; d\rho \; d\phi \; d\theta$$

13.7: Substitutions in multiple integrals (p. 1048)

Be able to perform a general coordinate transformation using the Jacobian and transform the region of integration appropriately. Here is the general procedure, it is laid out in more detail in the book:

1. Transform the boundary:
   - (a) Draw the region in the original coordinate system (usually cartesian)
   - (b) Write down the equation(s) for the boundary of the region, this could be a circle, ellipse, three lines to form a triangle, 4 lines to form a quadrilateral, etc.
   - (c) Plug in for x and y in terms of u and v in these boundary equations and solve the resulting equations for something you recognize (such as a line, parabola, ellipse, circle, etc.) and plot these new equation in the uv-plane to find your new region of integration
2. Transform the integral:
   - (a) $\iint_R f(x,y)\,dx\,dy = \iint_G f(x(u,v), y(u,v))\,|J(u,v)|\,du\,dv$
   - (b) The region G and your limits of integration come from the picture you drew in the uv-plane above
   - (c) You must compute the Jacobian $J(u,v)$ (p. 1048)
   - (d) Plug into you function f for x and y in terms of u and v

You should also be able to do this for 3d regions and coordinate transformations (see p. 1051).

Chapter 14

The exam will cover only sections 1 and 2 from Chapter 14 including line integrals, work, circulation and flux.

14.1: Line Integrals (p. 1061)

Key ideas:

1. Line integrals over smooth curves
2. Additivity of integrals and integrating over piecewise defined curves
3. Mass and moment formulas (see table 14.1, p. 1064)

$$\text{Given } C: \ \mathbf{r}(t) = g(t)\mathbf{i} + h(t)\mathbf{j} + k(t)\mathbf{k}, \ a \le t \le b$$
$$\int_C f(x,y,z)ds = \int_a^b f(g(t),h(t),k(t))|\mathbf{v}(t)|dt$$

You may also want to review parameterizing curves and be sure to note that we can write $ds = |\mathbf{v}(t)|dt$

14.2: Vector Fields, Work, Circulation, and Flux

Key ideas:

1. Vector fields, know what they are
2. Gradient fields, a special type of vector field defined as $\bigtriangledown f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$
3. Work: $W = \int_{t=a}^{t=b} \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} dt = \int_a^b M dx + N dy + P dz$
4. Flow/Circulation:
   (a) Circulation is just flow around a closed loop
   (b) Flow $= \int_a^b \mathbf{F} \cdot \mathbf{T} ds$ is evaluated the same way as work integrals
5. Flux:
   (a) Flux of $\mathbf{F}$ across $C = \int_C \mathbf{F} \cdot \mathbf{n} ds$
   (b) for a closed loop that moves counterclockwise once, Flux $= \oint_C M dy - N dx$

Although this material was kind of squeezed in before the break, don't neglect it as it is fair game for the test.

Finally, I can't stress enough that it is a VERY good idea to do some old exams once you've done some preliminary studying. This will give you some practive in applying your knowledge to harder problems which is the most common problem students have on the exams. Study hard, good luck and have a great break!