Projections and distances $\quad \text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$

Arc length, frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{|\,1+(f'(x))^2\,|^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{|\dot{x}^2+\dot{y}^2|^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

The Second Derivative Test

Suppose $f(x,y)$ and its first and second partial derivatives are continuous in a disk centered at (a,b), and $f_x(a,b) = f_y(a,b) = 0$. Let $D = f_{xx}f_{yy} - f_{xy}^2$.

1. If $D > 0$ and $f_{xx} < 0$ at (a,b), then f has a local maximum at (a,b).
2. If $D > 0$ and $f_{xx} > 0$ at (a,b), then f has a local minimum at (a,b).
3. If $D < 0$ at (a,b), then f has a saddle point at (a,b).
4. If $D = 0$ at (a,b), then the test is inconclusive.

Directional derivative, discriminant, and Lagrange multipliers

$$\frac{df}{ds} = (\nabla f)\cdot\mathbf{u} \qquad f_{xx}f_{yy} - (f_{xy})^2 \qquad \nabla f = \lambda\nabla g, \quad g = 0$$

Taylor's formula (at the point (x_0, y_0))

$$\begin{aligned} f(x,y) \;=\; & f(x_0,y_0) + \Big[(x-x_0)f_x(x_0,y_0) + (y-y_0)f_y(x_0,y_0)\Big] \\ & + \frac{1}{2!}\Big[(x-x_0)^2 f_{xx}(x_0,y_0) + 2(x-x_0)(y-y_0)f_{xy}(x_0,y_0) + (y-y_0)^2 f_{yy}(x_0,y_0)\Big] \\ & + \frac{1}{3!}\Big[(x-x_0)^3 f_{xxx}(x_0,y_0) + 3(x-x_0)^2(y-y_0)f_{xxy}(x_0,y_0) \\ & \qquad + 3(x-x_0)(y-y_0)^2 f_{xyy}(x_0,y_0) + (y-y_0)^3 f_{yyy}(x_0,y_0)\Big] + \cdots \end{aligned}$$

Linear approximation error

$$\big|E(x,y)\big| \le \frac{M}{2!}\big(|x-x_0| + |y-y_0|\big)^2, \qquad \text{where } \max\big\{|f_{xx}|, |f_{xy}|, |f_{yy}|\big\} \le M$$

Polar coordinates $\quad x = r\cos\theta \qquad y = r\sin\theta \qquad r^2 = x^2 + y^2 \qquad dA = dx\,dy = r\,dr\,d\theta$

Cylindrical and spherical coordinates

Cylindrical to Rectangular	Spherical to Cylindrical	Spherical to Rectangular
$x = r\cos\theta$	$r = \rho\sin\phi$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$z = \rho\cos\phi$	$y = \rho\sin\phi\sin\theta$
$z = z$	$\theta = \theta$	$z = \rho\cos\phi$

$$dV = dx\,dy\,dz = dz\,r\,d\theta\,dr = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Substitutions in multiple integrals

$$\iint_R f(x,y)\,dx\,dy = \iint_G f\big(x(u,v), y(u,v)\big)\,\big|J(u,v)\big|\,du\,dv \qquad \text{where} \qquad J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$$

Mass, moments, and center of mass $\quad$ Mass $\; M = \iint_R \delta\,dA$

Moments $\; M_x = \iint_R y\,\delta\,dA \qquad M_y = \iint_R x\,\delta\,dA \qquad$ Center of mass $\; \bar{x} = M_y/M \qquad \bar{y} = M_x/M$

Green's Theorem in a (x-y) plane (The curve C is traversed counterclockwise.)

$$\text{Circulation} = \oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \iint_R (\nabla\times\mathbf{F})\cdot\mathbf{k}\,dA = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\text{Outward Flux} = \oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R \nabla\cdot\mathbf{F}\,dA = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA$$

Surface area of level surface $g(x,y,z) = c \qquad S = \iint_S d\sigma = \iint_R \frac{|\nabla g|}{|\nabla g\cdot\mathbf{p}|}\,dA$

Stoke's Theorem $\quad \oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$

Divergence Theorem of Gauss $\quad \iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \nabla\cdot\mathbf{F}\,dV$