Name: ______________________

APPM 2350 Exam #1 Summer 2009

Be sure to include your name and a grading table on the front of your blue book. Also, include your name on this exam and submit with your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (20 points) Work the following problems. Not all of them are related to one another.

   (a) Determine if the line $x = 1 + 3t, \quad y = -2t, \quad z = -1 + t$ is parallel to the plane $x + 2y + z = -4$.

   (b) Find the distance between the line and the plane defined in part (a).

   (c) For a given vector $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$, find all vectors $\mathbf{w}$ such that $\mathbf{v} \times \mathbf{w} = \mathbf{w}$.

   (d) Show that if θ is the angle between $\mathbf{v}$ and $\mathbf{w}$, then $\tan(\theta) = \dfrac{|\mathbf{v} \times \mathbf{w}|}{\mathbf{v} \cdot \mathbf{w}}$ where $0 \le \theta < \dfrac{\pi}{2}$.

2. (25 points) A Romulan Warbird just uncloaked and fired directly at the Enterprise. You and Spock are thrown from your terminals in the explosion. When you get back to your post, you see that the Enterprise's primary computer has been badly damaged and all memory has been lost. You know that you were moving at a constant speed along a curve in space, however the specific trajectory has been lost. The backup computer quickly calculates that at the current time $\mathbf{r} = 2\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}$, $\mathbf{v} = 0\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{a} = 4\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$. Captain Kirk is attempting to execute evasive maneuvers and needs information. Using the above information, calculate the following values for him if you can. If you cannot, clearly state why there is insufficient information.

   (a) The unit tangent $\mathbf{T}$.

   (b) The unit normal $\mathbf{N}$.

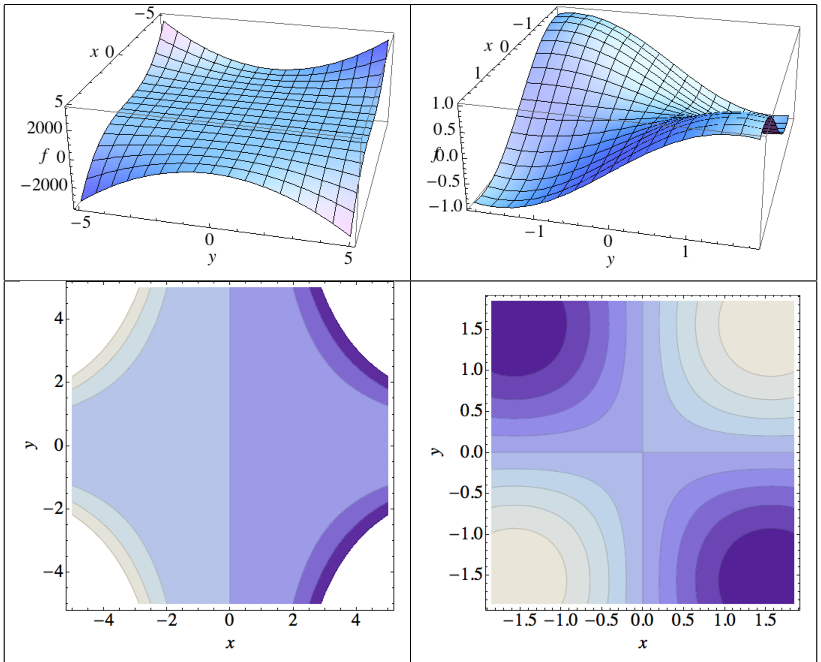
   (c) The curvature κ.

   (d) The torsion τ.

   (e) The value of $\mathbf{v} \cdot \mathbf{a}$ five seconds before impact with the Romulan plasma torpedo.

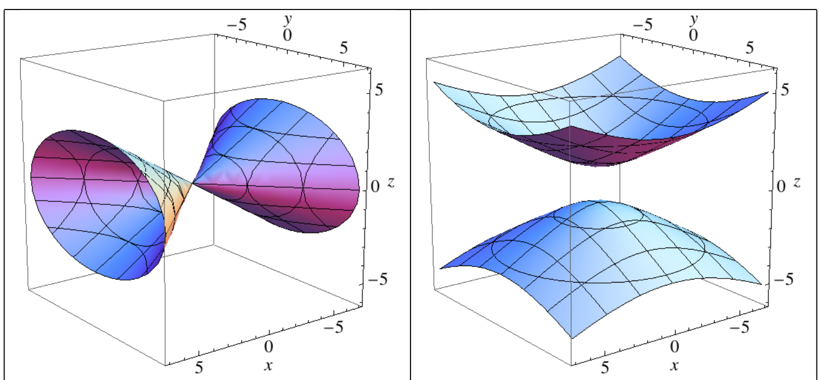
3. (30 points)

   (a) **Complete this problem directly on the exam sheet**. Each column in the figure on page 2 shows a surface and its associated family of level curves. A darker color corresponds to a smaller value for the level curve. Match each set of surfaces and associated level curves with one of the four possible functions (below each column) by circling the function. No justification is needed.



$f(x,y) = x^3y^2$	$f(x,y) = \sin^2(y)$
$f(x,y) = -x^2y^3$	$f(x,y) = \sin(x)\sin(y)$
$f(x,y) = -x^3y^2$	$f(x,y) = \cos(x)\cos(y)$
$f(x,y) = x^2y^3$	$f(x,y) = \cos(x)\sin(y)$

(b) **Complete this problem directly on the exam sheet**. Each column in the figure below shows a quadric surface. Match each surface with one of the three possible equations (below each column) by circling the equation. No justification is needed.



$-\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 - z^2 = 0$	$-\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 + z^2 = 1$
$-\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 + z^2 = 0$	$\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 - z^2 = 1$
$\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 - z^2 = 0$	$-\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 - z^2 = 1$

(c) Write an equation for an ellipsoid that is centered at the point (0,0,1) **and** where the axis parallel to the x-axis is half as long as the other two axes which have length one. **In addition** sketch the ellipsoid and label the axes and intercepts. *Hint: First write the equation for the ellipsoid when centered at the origin, then translate in z.*

4. (25 points) The Funtime Amusement Park has hired you to build a roller coaster. The specifications require that the ride be built in three parts. The ride begins at (1,0,0) and initially follow a circular, counter-clockwise path (when viewed looking towards $z = 0$ from positive z values) for a half circle to the point (-1,0,0). The second part of the ride follows a circular helix with radius 1 for two full rotations until the point (-1,0,8) is reached. Finally, the ride plunges to the origin in a straight line.

(a) Find a piecewise parameterization for each of the three smooth sections of the ride.

(b) Assume all units are in feet. How much rail does Funtime Amusement Park need to order in order to build the roller coaster?

(c) Future plans for the amusement park include a roller coaster that is an infinite helix that can be parameterized by the equation $\mathbf{r}(t) = a\cos(t)\mathbf{i} + a\sin(t)\mathbf{j} + bt\mathbf{k}$. Derive equations that express κ and τ in terms of a and b (the radius and climb-rate of the helix, respectively).

APPM 2350 — EXAM #1 EQUATIONS — SUMMER 2009

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$$

$$\tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N} = \frac{\begin{vmatrix}\dot{x} & \dot{y} & \dot{z}\\ \ddot{x} & \ddot{y} & \ddot{z}\\ \dddot{x} & \dddot{y} & \dddot{z}\end{vmatrix}}{|\mathbf{v}\times\mathbf{a}|^2}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$