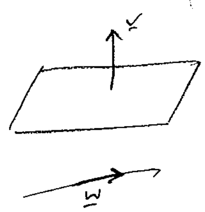
Problem 1

a) line points in direction of vector $\underline{w} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

plane has normal vector $\underline{v} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

IF parallel we have

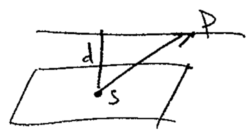


so $\underline{w}$ should be $\perp$ to $\underline{v}$

Indeed $\underline{v} \cdot \underline{w} = (3)(1) + (-2)(2) + (1)(1) = 0$ so line + plane are $\parallel$

b) Point on line $P(1, 0, -1)$

Point on Plane $S(0, 0, -4)$



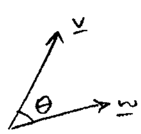
$$d = \left| \frac{\underline{v}}{|\underline{v}|} \cdot \overrightarrow{PS} \right| = \left| \frac{1}{\sqrt{6}} (\hat{\imath} + 2\hat{\jmath} + \hat{k}) \cdot (-\hat{\imath} + 0\hat{\jmath} - 3\hat{k}) \right|$$

$$= \left| \frac{1}{\sqrt{6}} (-1 + 0 - 3) \right| = \frac{4}{\sqrt{6}}$$

c) The cross product of $\underline{v} \times \underline{w}$ results in a vector that is $\perp$ to both $\underline{v}$ and $\underline{w}$.

Therefore only $\underline{w} = \underline{0} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ satisfies the equality $\underline{v} \times \underline{w} = \underline{w}$

d) $\underline{v} \cdot \underline{w} = |\underline{v}||\underline{w}| \cos\theta$



$\underline{v} \times \underline{w} = |\underline{v}||\underline{w}| \sin\theta \, \hat{n}$

$\Rightarrow |\underline{v} \times \underline{w}| = |\underline{v}||\underline{w}| \sin\theta$

so $$\frac{|\underline{v} \times \underline{w}|}{\underline{v} \cdot \underline{w}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Problem 2

a) $\hat{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{4}{5}\hat{j} + \dfrac{3}{5}\hat{k}$

b) $\hat{N} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$ since acceleration is $\perp$ to velocity due to constant speed

$= \hat{i}$

c) $K = \dfrac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$ where $\mathbf{v}\times\mathbf{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 4 & 3 \\ 4 & 0 & 0 \end{vmatrix}$

$= 0\hat{i} - (-12)\hat{j} - 16\hat{k} = 12\hat{j} - 16\hat{k}$

$\Rightarrow |\mathbf{v}\times\mathbf{a}| = \sqrt{12^2 + (16)^2} = \sqrt{144 + 256} = \sqrt{400}$

$= 20$

$\Rightarrow K = \dfrac{20}{(5)^3} = \dfrac{20}{125} = \dfrac{4}{25}$

d) $\tau = \dfrac{\begin{pmatrix} \longleftarrow v \longrightarrow \\ \longleftarrow a \longrightarrow \\ \longleftarrow \dot{a} \longrightarrow \end{pmatrix}}{|\mathbf{v}\times\mathbf{a}|^2}$ but we can't compute $\dot{a}$ w/ out a Functional form For $a(t)$.

e.) Since we're told we were moving at a constant velocity, the acceleration must be $\perp$ to the velocity, hence $\mathbf{v}\cdot\mathbf{a} = 0$.

Problem 3

a) left column: $f(x,y) = -x^3y^2$

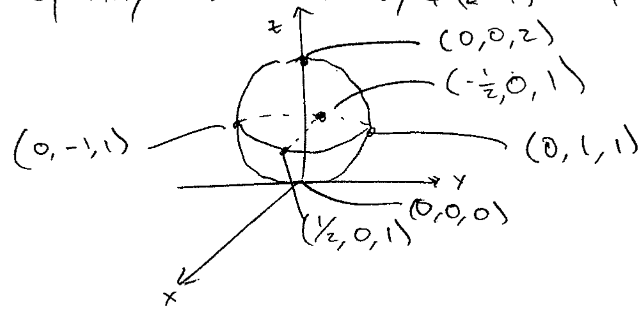
right column: $f(x,y) = \sin(x)\sin(y)$

b) left column: $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 - z^2 = 0$

right column: $-\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 + z^2 = 1$

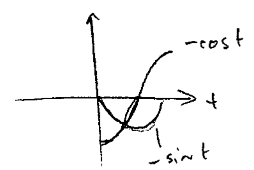
c) eq'n for ellipse centered @ origin $\left(\frac{x}{1/2}\right)^2 + y^2 + z^2 = 1$

so desired equality is $4x^2 + y^2 + (z-1)^2 = 1$



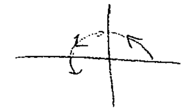
Problem 4

a) Part 1: $\underline{r}_1(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$ for $t \in [0,\pi]$



Part 2: $\underline{r}_2(t) = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath} + \alpha t$ for $t \in [0,4\pi]$

since $t = 4\pi$ chosen to complete 2 revolutions, we reach a height of $4\pi\alpha$. Since we want to this height to equal 8 we have $\alpha = \frac{2}{\pi}$



Part 3: line points along vector $\overrightarrow{PO} = (0-(-1))\hat{\imath} + (0-0)\hat{\jmath} + (0-8)\hat{k}$

so lin is
$$x(t) = t-1$$
$$y(t) = 0$$
$$z(t) = -8t+8$$

$\Rightarrow \underline{r}_3(t) = (t-1)\hat{\imath} + 0\hat{\jmath} + (8-8t)\hat{k}$ for $t \in [0,1]$

b) Length of part $i = s_i$ requires knowing $|\underline{v}_i|$

so
$$\underline{v}_1 = -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} \Rightarrow |\underline{v}_1| = 1$$

$$\underline{v}_2 = \sin t\,\hat{\imath} - \cos t\,\hat{\jmath} + \frac{2}{\pi}\hat{k} \Rightarrow |\underline{v}_2| = \sqrt{1 + \frac{4}{\pi^2}}$$

$$\underline{v}_3 = -\hat{\imath} - 8\hat{k} \Rightarrow |\underline{v}_3| = \sqrt{1^2 + (-8)^2} = \sqrt{65}$$

$$\Rightarrow s_1 = \int_0^{\pi} |\underline{v}_1|\,dt = \pi$$

$$s_2 = \int_0^{4\pi} \sqrt{1 + \frac{4}{\pi^2}}\,dt = 4\pi\sqrt{1 + \frac{4}{\pi^2}}$$

$$s_3 = \int_0^1 \sqrt{65}\,dt = \sqrt{65}$$

$$\Rightarrow \text{total length is } \left(\pi + 4\pi\sqrt{1 + \frac{4}{\pi^2}} + \sqrt{65}\right) \text{ ft}$$

c)
$$\underline{r}(t) = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + bt\,\hat{k}$$
$$\underline{v}(t) = -a\sin t\,\hat{\imath} + a\cos t\,\hat{\jmath} + b\,\hat{k}$$

$$\underline{a}(t) = -a\cos t\,\hat{\imath} - a\sin t\,\hat{\jmath}$$
$$\dot{\underline{a}}(t) = a\sin t\,\hat{\imath} - a\cos t\,\hat{\jmath}$$
$$\Rightarrow |\underline{v}| = \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2} = \sqrt{a^2 + b^2}$$

$$\underline{v} \times \underline{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -a\sin t & a\cos t & b \\ -a\cos t & -a\sin t & 0 \end{vmatrix} = \hat{\imath}(0 + ab\sin t) - \hat{\jmath}(0 + ab\cos t) + \hat{k}(a^2\sin^2 t + a^2\cos^2 t)$$

$$= ab\sin t\,\hat{\imath} - ab\cos t\,\hat{\jmath} + a^2\hat{k}$$

$$\Rightarrow |\underline{v} \times \underline{a}| = \sqrt{a^2b^2\sin^2 t + a^2b^2\cos^2 t + a^4}$$

$$= \sqrt{a^2b^2 + a^4} = a\sqrt{a^2 + b^2}$$

$$\Rightarrow \kappa = \frac{a\sqrt{a^2 + b^2}}{(a^2 + b^2)^{3/2}} = \frac{a}{a^2 + b^2}$$

and

$$\tau = \frac{\begin{vmatrix} -a\sin t & a\cos t & b \\ -a\cos t & -a\sin t & 0 \\ a\sin t & -a\cos t & 0 \end{vmatrix}}{|\underline{v} \times \underline{a}|^2}$$

The numerator evaluates to

$$-a\sin t(0-0) - a\cos t(0-0) + b(a^2\cos^2 t + a^2\sin^2 t) = a^2 b$$

so

$$\tau = \frac{a^2 b}{\left(a\sqrt{a^2+b^2}\right)^2} = \frac{b}{a^2+b^2}$$