

Name: _____

APPM 2350

Exam #2

Summer 2009

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (20 points) The following questions are not related to one another.
 - (a) What is the equation for the tangent plane to the surface $x + y + z = 1$ at the point $P_0(0, 1, 0)$?
 - (b) By about how much will $f(x, y, z) = e^x \cos(yz)$ change as the point $P(x, y, z)$ moves from the origin a distance of $ds = 0.1$ units in the direction of $2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$?
 - (c) Verify that the function $w(x, t) = \cos(2x + 2ct)$ satisfies the one-dimensional wave equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

where c is a constant and represents the velocity of the wave. It must be clear from your work that $w(x, t)$ is a solution to the wave equation in order to receive full credit for this problem.

2. (20 points) Let $f(x, y) = x^2 - 3xy + 5$.
 - (a) Determine the linearization of $f(x, y)$ at the point $(2, 1)$.
 - (b) Bound the error in this approximation if $-\frac{1}{10} \leq x - 2 \leq \frac{1}{10}$ and $-\frac{1}{10} \leq y - 1 \leq \frac{1}{10}$.
 - (c) What is the quadratic (second order) approximation to $f(x, y)$ at the point $(2, 1)$.
3. (20 points) Consider a right-circular cylinder where both the radius and height are changing with respect to time.
 - (a) Find the equation for the change in volume of the cylinder with respect to time.
 - (b) If the radius, r , is increasing at the rate of $3 \frac{\text{in}}{\text{min}}$ and its height, h , is decreasing at the rate of $5 \frac{\text{in}}{\text{min}}$. At what rate is the volume of the cylinder changing when the radius is 10 inches and the height is 8 inches?

4. (20 points) Let S be a surface that is the graph of $f(x, y) = 10 - x^2 - y^2$. Suppose the radiation in space at each point (x, y, z) is $R(x, y, z) = x^2y + y^2z + 4x + 4y + z$.
- (a) Compute a normal vector to the surface S at the point $(0, 0, 10)$.
 - (b) Compute the gradient of $R(x, y, z)$.
 - (c) What is the equation for the tangent plane to the surface at the point $(0, 0, 10)$?
 - (d) Among all the possible directions tangent to the surface S at the point $(0, 0, 10)$, which direction will make the rate of change of radiation at $(0, 0, 10)$ a minimum?
5. (20 points) The temperature of a metal plate is given by $T(x, y) = x^2 - y^2 - x$.
- (a) Identify and classify all critical points of $T(x, y)$.
 - (b) If the metal plate is bounded by the lines $y = 0$, $x = 0$ and $x + y = 1$, determine all possible extremizing points of $T(x, y)$ on the plate. It is not necessary to evaluate $T(x, y)$ at these points.
 - (c) If the metal plate is cut into the shape described by $x^2 + y^2 \leq 1$, find all possible extremizing values of $T(x, y)$ on the plate. It is not necessary to evaluate $T(x, y)$ at these points.
6. Bonus (1 point each)
- (a) Who is the original guitarist that played a guitar solo on "Beat It"?

Slash Eddie Van Halen Lenny Kravitz Carlos Santana Eric Clapton
 - (b) For how many years was Farrah Fawcett a regular on Charlie's Angels?
 - (c) How old was Farrah Fawcett when she last appeared in Playboy magazine?
 - (d) For one bonus point, after you finish the exam, come to the front of the room and do the Moonwalk.