Name: _____

APPM 2350 Exam #3 Summer 2009

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please start each new problem **on a new page**. Good luck!

1. (25 points) Not all of the following questions relate to one another.

   (a) Consider the point (π, π, π) in cylindrical coordinates.

      i. Convert the point into rectangular coordinates.

      ii. Convert the point into spherical coordinates.

   (b) Convert $2x^2 + 2y^2 + 2z^2 = 1$ to spherical.

   (c) Convert $\rho^2 \sin\phi \cos\phi \cos\theta = 1$ to rectangular.

   (d) Consider a thin plate bounded by $y = 0$, $y = x^2$, and $x = 1$ with density $\delta(x, y) = 3x$.

      i. Sketch the region.

      ii. Find the mass of the plate.

      iii. Find the centroid of the plate.

   (e) Consider the region bounded by $y = \sqrt{4 - x^2}$, $y = \sqrt{1 - x^2}$, and $y = 0$.

      i. Sketch the region.

      ii. Set up the double integral for finding the area of the region in polar coordinates.

      iii. Evaluate the double integral.

97% of all statistics are made up.

2. (25 points)Consider the following integral

$$\int_{\frac{-\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{-\sqrt{x^2+y^2}} dz\, dy\, dx$$

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   (a) Sketch the region of integration, including all boundaries. Also, describe in words what surfaces bound the region.

   (b) Write the integral using $dz\, dr\, d\theta$.

   (c) Write the integral using the order $dr\, dz\, d\theta$.

   (d) Write the integral in spherical coordinates using the order $d\rho\, d\phi\, d\theta$.

   (e) Evaluate one of the integrals from (b)-(d).

3. (30 points) Researchers studying global warming are using new parabolic rain gauges to measure changing rain patterns. These parabolic gauges are 2 units high and are in the shape of the graph $z = x^2 + y^2$.

   (a) Sketch the parabolic rain gauge.

   (b) Set up, but do not evaluate, an integral to calculate the total volume of rain the gauge can hold using the order $dzdydx$.

   (c) Set up and evaluate an integral to calculate the volume of rain held by the gauge if it is filled to an arbitrary height $0 \leq h \leq 2$.

4. (25 points) Consider the integral

$$\iint_R y^3 \cos(2x - y) dy\, dx$$

   where R is the region bounded by the parallelogram with vertices (0,0), (2,0) (3,2) and (1,2). Assume the transformation is given by $x = \frac{1}{2}(u + v)$ and $y = v$.

   (a) Draw the region R in the xy-plane. Indicate all vertices.

   (b) Draw the region S in the uv-plane. Indicate all vertices.

   (c) Compute the Jacobian $J(u, v)$.

   (d) Rewrite the integral given above as an integral over S in the uv-plane.

   (e) Compute the integral in part (d).