Name: ______________________________

APPM 2350 FINAL Exam Summer 2009

Be sure to include your name and a grading table on the front of your blue book. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please start of each new problem **on a new page**. Good luck! Note that this exam is worth 150 points.

1. (15 points) Indicate whether the question is true or false. No justification needed.

   (a) The vectors $< -2, 3 >$ and $< 4, 6 >$ are orthogonal.

   (b) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ then $\mathbf{b} = \mathbf{c}$.

   (c) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$.

   (d) If $\mathbf{r}$ is differentiable and $|\mathbf{r}|$ is constant, $\mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = 0$.

   (e) If $f(x, y) = x^2 e^y$, $x(t) = t^2 - 1$, and $y(t) = \sin(t)$, then

   $$\frac{df}{dt} = 2(t^2 - 1)(2t) + (t^2 - 1)^2 \cos(t)$$

2. (20 points) Set up, but do not solve, a system of equations that will find the extrema of $f(x, y, z) = x^2 + y^2 + z^2$ on the intersection of $xyz = 1$ and $x^2 + y^2 + 2z^2 = 4$ using Lagrange Multipliers. Note, the extrema will satisfy $\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$. Indicate all n equations <u>and</u> n unknowns in the resulting system.

3. (30 points) Consider the surface described by $x = 1 - y^2 - 2z^2$ above the yz-plane and a vector field $\mathbf{F} = x\hat{\mathbf{i}} + (y + 1)\hat{\mathbf{j}} + yz^2\hat{\mathbf{k}}$.

   (a) Name and sketch this quadric surface. Clearly indicate the intercepts.

   (b) Sketch the curve C that represents the boundary of the surface. Be sure to label your axes and clearly indicate intercepts.

   (c) Set up **but do not evaluate** the line integral for calculating the circulation about the curve C when traversed in a **clockwise** manner as viewed from positive x values.

   (d) Set up **but do not evaluate** a surface integral to compute the same circulation. Go as far as to determine R, the shadow or projection of the surface. Write the surface integral as a double integral over this region. Be sure to include the correct limits of integration.

4. (20 points) Consider the function $f(x, y) = 2x^2 - 4xy + y^3 + 2$.

   (a) Find and classify all critical point(s) of $f(x, y)$.

   (b) Find the **second order** Taylor expansion of $f(x, y)$ about the point $(0, 0)$.

   (c) What is an upper bound for the error in the **first order** Taylor expansion about the origin when evaluated at the point $(1, 1)$?

5. (20 points) Calculate the amount of work done by the conservative force field

$$\mathbf{F}(x, y, z) =< 3x^2y^2z, 2x^3yz, x^3y^2 - e^{-z} >$$

   along the line segments from $(1, 0, -1)$ to $(0, 0, -1)$, then from $(0, 0, -1)$ to $(0, -1, -1)$, and finally from $(0, -1, -1)$ to $(0, -1, 1)$.

6. (25 points) Researchers have discovered that the rain gauges described in Exam 3, the "Rain Gauge 2000," tend to blow away in high winds due to their shape. As a result, they are considering using a rain gauge in the shape of $z = 2\sqrt{x^2 + y^2}$ known as the "Precip Pal Mark V." They do not yet know whether they will purchase the "Rain Gauge 2000" or the "Precip Pal Mark V" and the deciding factor depends on the volume that the "Precip Pal Mark V" contains if filled to a height h with rainwater.

   (a) Sketch the "Precip Pal Mark V" and indicate the height h.

   (b) Set up, **but do not evaluate**, an integral to calculate the volume of rain the gauge can hold when filled to an arbitrary height h using the order $dzdydx$.

   (c) Set up **but do not evaluate** an integral to calculate the volume of rain held by the gauge if it is filled to an arbitrary height h using the order $dzdrd\theta$.

7. (20 points) Find the outward flux of $\mathbf{F} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ over the cube $0 \leq x \leq 2$, $0 \leq y \leq 2$, and $0 \leq z \leq 2$.

Why did the chicken cross the Mobius strip?
To get to the same side.