APPM 2350 **EXAM #2** **Summer 2011**

On the front of your bluebook write: (1) your name, (2) your student ID number, (3) your instructor's name, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (a) What is the general form of the vector always orthogonal to the surface defined by $z = f(x, y)$?
   (b) What is the total differential for the volume $V(r, h) = \frac{\pi r^2 h}{3} + \frac{2}{3}\pi r^3$ of a cone and half sphere?
   (c) An ant is constrained to walk on a surface defined by $F(x, y, z) = 0$. Let T be the tangent vector of the ant's motion on this surface. What is $\nabla F \cdot T$?
   (d) If $f(x, y) = x^3 + y^3$ and $x(s) = \sin(s)$ and $y(s) = \cos(s)$ then compute $\frac{df}{ds}$.

2. Two velociraptors are hunting you in a 16 by 12 room. They always hunt in pairs, one to distract you from the front while the other sneaks up from the side. You are at $(0, 0)$, the velociraptors are at $(0, 5)$ and $(6, 0)$, and there is a door in the corner at (-8,6). The likelihood you will get eaten is given by $P(x, y) = ((x - 6)^2 + y^2)^{-1} + (x^2 + (y - 5)^2)^{-1}$.
   (a) What direction should you go in to minimize your chance of getting devoured?
   (b) If you move at a speed of 0.5 m/s in this direction, how is your chance of being eaten changing in time? (i.e. What is $\frac{dP}{dt}$?)
   (c) If you run toward the door, how is your chance of being eaten changing as a function of distance? (i.e. What is $\frac{dP}{ds}$?)

3. Consider the function $f(x, y) = (x-1)^2-2y^2+\frac{1}{2}x+y$ on the domain $D : \{(x, y)|0 \le x, 0 \le y, x+y < 2\}$
   (a) Locate and classify all the critical points on the interior of the domain.
   (b) Identify the (x, y) locations of the absolute minimum and absolute maximum values of the surface. (Hint: Be mindful of the domain boundary)
   (c) Consider the surface descfibed by $z = f(x, y)$, compute the unit surface normal at the point $(\frac{1}{2}, \frac{1}{2})$

4. A lunar shuttle has just landed on the surface of the moon. Take the landing site to be the origin. The crew needs to set up base camp exactly 1 mile away in any direction. Prior to landing, the ship surveyed the solar light intensity of the ground below. The intensity can be well described by the function $S(x, y) = x^2 + y + 1$.
   (a) Draw a picture of the landing site, and contours that represent the light intensity (draw at least 4 contours).
   (b) On the same diagram draw a curve that represents all possible base camp locations.
   (c) In what direction can the crew head from the landing site to find a base camp location with maximal sunlight?
   (d) Once the camp is set up the crew launches a robot with velocity $\langle 3, 2\rangle$. What is the time rate change of light intensity that the robot experiences? (i.e. What is $\frac{dS}{dt}$?)

Projections and distances $\quad \text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$

Arc length, frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{|\,1+(f'(x))^2\,|^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{|\dot{x}^2+\dot{y}^2|^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Directional derivative, discriminant, and Lagrange multipliers

$$\frac{df}{ds} = (\nabla f)\cdot\mathbf{u} \qquad f_{xx}f_{yy} - (f_{xy})^2 \qquad \nabla f = \lambda\nabla g, \quad g = 0$$