APPM 2350 — BASIC TABLE

The Frenet Formulas

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}, \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}, \qquad \frac{d\mathbf{T}}{ds} = \kappa\,\mathbf{N}, \qquad \frac{d\mathbf{B}}{ds} = -\tau\,\mathbf{N}.$$

The Decomposition of Acceleration

$$\mathbf{a}(t) = a_T\mathbf{T} + a_N\mathbf{N}, \qquad a_T = \frac{d|\mathbf{v}|}{dt}, \quad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Linear Approximations

$$\text{Error } |E(x,y)| \le \frac{1}{2}M(|x-x_0| + |y-y_0|)^2, \qquad \text{where } |f_{xx}|, |f_{xy}|, |f_{yx}|, |f_{yy}| \le M$$

Second Derivative Test

Suppose f_x and f_y are continuous in a disk centered at (a,b) and $f_x(a,b) = f_y(a,b) = 0$.

1. If $f_{xx}f_{yy} - f_{xy}^2 > 0$ and $f_{xx} < 0$ at (a,b), then f has a local maximum at (a,b);
2. If $f_{xx}f_{yy} - f_{xy}^2 > 0$ and $f_{xx} > 0$ at (a,b), then f has a local minimum at (a,b);
3. If $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a,b), then f has a saddle at (a,b).

Lagrange Multipliers

$$\nabla f = \lambda\nabla g, \qquad g = 0$$

Taylor's Formula (about $x = 0$, $y = 0$)

$$f(0,0) + \{f_x(0,0)x + f_y(0,0)y\} + \frac{1}{2!}\left\{f_{xx}(0,0)x^2 + 2f_{xy}(0,0)xy + f_{yy}(0,0)y^2\right\} + \cdots$$

Moments and Centers of Mass (for three dimensional solids)

Center of mass

$$\begin{aligned}\bar{x} &= \frac{M_{yz}}{M} = \frac{1}{M}\iiint_R x\delta(x,y,z)\,dV,\\ \bar{y} &= \frac{M_{zx}}{M} = \frac{1}{M}\iiint_R y\delta(x,y,z)\,dV,\\ \bar{z} &= \frac{M_{xy}}{M} = \frac{1}{M}\iiint_R z\delta(x,y,z)\,dV.\end{aligned}$$

Moments of Inertia

$$I_x = \iiint_R (y^2+z^2)\delta(x,y,z)\,dV,\ I_y = \iiint_R (z^2+x^2)\delta(x,y,z)\,dV,\ I_z = \iiint_R (x^2+y^2)\delta(x,y,z)\,dV$$

Radii of Gyration

$$R_x = \sqrt{I_x/M}, \qquad R_y = \sqrt{I_y/M}, \qquad R_z = \sqrt{I_z/M}$$

Polar Coordinates

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad r^2 = x^2 + y^2, \qquad dA = dx\,dy = r\,dr\,d\theta$$

Spherical and Cylindrical Coordinates

Cylindrical to Rectangular	Spherical to Cylindrical	Spherical to Rectangular
$x = r\cos\theta$	$r = \rho\sin\phi$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$z = \rho\cos\phi$	$y = \rho\sin\phi\sin\theta$
$z = z$	$\theta = \theta$	$z = \rho\cos\phi$

$$dV = dx\,dy\,dz = dz\,r\,dr\,d\theta = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Substitutions in Multiple Integrals

$$\int\!\!\int_R f(x,y)\,dx\,dy = \int\!\!\int_G f(x(u,v),y(u,v))|J(u,v)|\,du\,dv, \qquad J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$$

Green's Theorem

$$\begin{aligned} \text{Flux} &= \oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \oint_C M\,dy - N\,dx = \int\!\!\int_R \operatorname{div}\mathbf{F}\,dA = \int\!\!\int_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA \\ \text{Circulation} &= \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C M\,dx + N\,dy = \int\!\!\int_R \mathbf{curl}\,\mathbf{F}\cdot\mathbf{k}\,dA = \int\!\!\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA \end{aligned}$$

Surface Integrals

$$\begin{aligned} d\sigma &= \sqrt{1 + h_x^2 + h_y^2}\;dA \qquad \text{for the surface } z = h(x,y), \\ &= \frac{|\nabla f|}{|\nabla f\cdot\mathbf{p}|}\,dA \qquad \text{for the surface } f(x,y,z) = c, \end{aligned}$$

$$\text{Surface area } = \int\!\!\int_S d\sigma, \quad \text{Flux} = \int\!\!\int_S \mathbf{F}\cdot\mathbf{n}\,d\sigma$$

Stokes's Theorem

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \int\!\!\int_S \mathbf{curl}\ \mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma$$

The Divergence Theorem (Gauss's Theorem)

$$\int\!\!\int_S \mathbf{F}\cdot\hat{\mathbf{n}}\,d\sigma = \int\!\!\int\!\!\int_D \operatorname{div}\mathbf{F}\,dV$$