

Using EULER'S METHOD to solve $y' = y - t, y(0) = 0$, to get $y(1)$.

Stepsize $h = 0.5$

n	t_n	$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$	$f(t_n, y_n) = y_n - t_n$	$y(t_n)$ (exact solution)
0	0	0	$0 - 0 = 0$	0
1	0.5	$0 + 0.5 \times 0 = 0$	$0 - 0.5 = -0.5$	-0.1487
2	1.0	$0 + 0.5 \times (-0.5) = -0.25$		-0.7183

Stepsize $h = 0.25$

n	t_n	$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$	$f(t_n, y_n) = y_n - t_n$	$y(t_n)$ (exact solution)
0	0	0	$0 - 0 = 0$	0
1	0.25	$0 + 0.25 \times 0 = 0$	$0 - 0.25 = -0.25$	-0.0340
2	0.5	$0 + 0.25 \times (-0.25) = -0.0625$	$-0.0625 - 0.5 = -0.5625$	-0.1487
3	0.75	$-0.0625 + 0.25 \times (-0.5625) = -0.2031$	$-0.2031 - 0.75 = -0.9531$	-0.367
4	1.0	$-0.2031 + 0.25 \times (-0.9531) = -0.4414$		-0.7183

Using EULER'S METHOD to solve $y' = 2y, y(0) = 1$, to get $y(2)$.

Stepsize $h = 0.5$

n	t_n	$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$	$f(t_n, y_n) = 2y_n$	$y(t_n)$ (exact solution)
0	0	1	$2 \times 1 = 2$	1
1	0.5	$1 + 0.5 \times 2 = 2$	$2 \times 2 = 4$	2.7183
2	1.0	$2 + 0.5 \times 4 = 4$	$2 \times 4 = 8$	7.3891
3	1.5	$4 + 0.5 \times 8 = 8$	$2 \times 8 = 16$	20.0855
4	2.0	$8 + 0.5 \times 16 = 16$		54.5982

Stepsize $h = 0.25$

n	t_n	$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$	$f(t_n, y_n) = 2y_n$	$y(t_n)$ (exact solution)
0	0	1	$2 \times 1 = 2$	1
1	0.25	$1 + 0.25 \times 2 = 1.5$	$2 \times 1.5 = 3$	1.6487
2	0.5	$1.5 + 0.25 \times 3 = 2.25$	$2 \times 2.25 = 4.5$	2.7183
3	0.75	$2.25 + 0.25 \times 4.5 = 3.375$	$2 \times 3.375 = 6.75$	4.4817
4	1.0	$3.375 + 0.25 \times 6.75 = 5.0625$	$2 \times 5.0625 = 10.125$	7.3891
5	1.25	$5.0625 + 0.25 \times 10.125 = 7.5938$	$2 \times 7.5938 = 15.1875$	12.1825
6	1.5	$7.5938 + 0.25 \times 15.1875 = 11.3906$	$2 \times 11.3906 = 22.7813$	20.0855
7	1.75	$11.3906 + 0.25 \times 22.7813 = 17.0859$	$2 \times 17.0859 = 34.1719$	33.1155
8	2.0	$17.0859 + 0.25 \times 34.1719 = 25.6289$		54.5982

The error using EULER'S METHOD and the RK4 METHOD to solve $y' = y - t, y(0) = 0$, to get $y(1)$. The "ratio" column shows the ratio between the previous error (for twice the stepsize) and the current error.

h	Euler error	ratio	RK4 error	ratio
0.50000	0.468281828459	-----	0.000935637053	-----
0.25000	0.276875578459	1.6913	0.000071889258	13.0150
0.12500	0.152497314509	1.8156	0.000004984042	14.4239
0.06250	0.080353331092	1.8978	0.000000328118	15.1898
0.03125	0.041291699081	1.9460	0.000000021048	15.5892
0.01562	0.020936875894	1.9722	0.000000001333	15.7931
0.00781	0.010542808771	1.9859	0.000000000084	15.8961
0.00391	0.005290204206	1.9929	0.000000000005	15.9471
0.00195	0.002649828290	1.9964	0.000000000000	15.9818
0.00098	0.001326098993	1.9982	0.000000000000	15.4323

As above but for $y' = 2y, y(0) = 1$, to get $y(2)$.

h	Euler error	ratio	RK4 error	ratio
0.50000	38.598150033144	-----	0.794906278322	-----
0.25000	28.969243783144	1.3324	0.075132288169	10.5801
0.12500	19.071013245139	1.5190	0.005775508243	13.0088
0.06250	11.258967205645	1.6939	0.000400427562	14.4234
0.03125	6.175106995122	1.8233	0.000026361737	15.1897
0.01562	3.242636135123	1.9043	0.000001691030	15.5892
0.00781	1.662779094503	1.9501	0.000000107074	15.7931
0.00391	0.842116830885	1.9745	0.000000006736	15.8962
0.00195	0.423787224334	1.9871	0.000000000422	15.9474
0.00098	0.212581800563	1.9935	0.000000000026	15.9753