ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Start each problem on a **new page**. Show ALL of your work in your bluebook and **box in your final answer**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and electronic devices of ANY sort are NOT permitted. One $8'' \times 11''$, two-sided, sheet of notes is allowed.

1. (20 points)

   (a) Find the general solution of $y'' - 2y' - 3y = 0$.

   (b) Using a similar method, find the solution of the Initial Value Problem $y''' - y' = 0$, $y(0) = 2$, $y'(0) = 0$, $y''(0) = 1$.

2. (20 points) Let $\vec{u}_1$ and $\vec{u}_2$ be the vectors in $\mathbb{R}^3$ given by

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

   (a) Show that $\vec{u}_1$ and $\vec{u}_2$ are linearly independent.

   (b) Find a vector $\vec{u}_3$ that is NOT in $\text{Span}(\vec{u}_1, \vec{u}_2)$. Justify your answer.

   (c) Construct a basis for $\mathbb{R}^3$ that includes $\vec{u}_1$ and $\vec{u}_2$.

3. (20 points) (a) Use elementary row operations to determine the values of λ for which the following system of equations is consistent:

$$\begin{aligned} x_1 + x_3 &= 2, \\ 2x_1 - x_2 &= 0, \\ -x_1 + x_2 + x_3 &= \lambda. \end{aligned}$$

   (b) For the value(s) of λ for which it is consistent, how many solutions are there?

4. (20 points) Determine whether or not the following sets are vector spaces. The usual addition and multiplication rules are assumed. Justify your answers.

   (a) Polynomials with only even powers of x (for example, $p(x) = 3 + 2x^2 - x^6 + 7x^8$).

   (b) The solutions of $y'' + y = 0$.

   (c) The solutions of $y' - 2y = 1$.

   (d) The set of all 7×7 matrices A such that $\det(A) = 1$.

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5. (20 points)

   (a) Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. Find or explain why it is not defined:

   - BA,
   - $(BA)^{-1}$,
   - $|B|$,
   - A^T.

   (b) Is there a matrix C such that $C^2 = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ and $C^3 = \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix}$? Explain your answer.
   (Hint: use the properties of determinants.)