APPM 2360: LAB 1
An Inconvenient Truth: Lake water contaminant analysis

http://www.freefoto.com/images/13/08/13_08_52—Industry-Liquid-Pollution_web.jpg

Introduction

According to the Environmental Protection Agency,

> Pollutants discharged upstream often become the problem of someone who lives downstream (or of the aquatic life that exists instream), and all of the activities that take place in a watershed can have a water quality impact elsewhere in the watershed...Forty-seven states reported causes of impairment in their lakes. Overall, these states reported that metals and nutrients are the most common causes of nonsupport in assessed lakes, affecting 47% and 40% of impaired lake acres, respectively...Arsenic occurs naturally in rocks and soil, water, air, and plants and animals. It can be further released into the environment through natural activities such as volcanic action, erosion of rocks, and forest fires, or through human actions. Approximately 90 percent of industrial arsenic in the U.S. is currently used as a wood preservative, but arsenic is also used in paints, dyes, metals, drugs, soaps, and semi-conductors. Agricultural applications, mining, and smelting also contribute to arsenic releases in the environment.[1]

Lab Goals and Instructions

In this lab, we will model the concentration of arsenic in a fresh water lake. This lab is designed to demonstrate the usefulness of some basic numerical and qualitative techniques. There are topics at the end of the lab which need to be addressed in the lab write-up.

Labs may be done in groups of 3 or less. One report must be turned in for each group. Labs must include each student's:

- name
- student number
- section number
- Professor's name
- TA's name

This lab is due on Thursdsay, October 8^{th}, 2009. **Late labs will not be accepted.** A paper copy of your group's lab must be turned in at recitation and **one** member of your group must upload an electronic copy of your lab to SafeAssign via CULearn. Detailed instructions on electronic submission can be found here. Any group which does not submit an electronic copy will receive a grade of 0. Labs will be returned in recitation to the person whose name is first on the report. Lab regrades must be given to your recitation teacher no later than the week following the return of the labs.

[1] EPA

Simply answering the lab questions will not earn you a good grade. To fare well, you will need to pay close attention to the formatting guidelines. As much as 20% of your lab grade is based on formatting. Any questions regarding the lab policies may be addressed to the lab course coordinator, Ted Galanthay.

Model

Consider a stream running into a lake of constant volume V. Arsenic is discharged into the stream from a nearby factory. At time t, the lake contains an amount $Q(t)$ (in micrograms) of arsenic evenly distributed throughout the lake with concentration $c(t)$ (in micrograms per liter), where $c(t) = Q(t)/V$. Assume that water containing a concentration k of arsenic enters the lake at a rate r, and that water leaves at the same rate. Suppose that arsenic is also added directly to the lake at a constant rate P.

http://feww.files.wordpress.com/2007/10/water_pollution.jpg

The model for the amount of arsenic in the lake is derived by using the "Balance Principle":

Change in the amount of pollutant = Amount flowing in + Amount added - Amount flowing out

It is written mathematically as:

$$Vc'(t) = kr + P - c(t)r \tag{1}$$

Let's begin with the following parameter values:

$k = 75$	The concentration of the arsenic (μ g/liter)
$P = 0$	No arsenic is directly added to the lake
$V = 8 \times 10^{12}$	Volume of the lake (liters)
$r = 6.44 \times 10^9$	Flow rate (liters per day)

In deriving the mathematical model(1) we make the assumption that the concentration of the arsenic coming into the lake is a constant. However, now suppose that the concentration of arsenic fluctuates on a daily basis due to the factory's shift operations (more people work during the day shift than on the night shift), and also on a weekly basis due to production cycles. We add two periodic terms to take care of the two fluctuations.

Thus, the expression for the concentration of arsenic flowing into the lake is modified to be:

$$k + a * \sin(2\pi t) + b * \sin(2\pi t/7)$$

where $a * \sin(2\pi t)$ represents the daily fluctuation and $b * \sin(2\pi t/7)$ represents the weekly fluctuation. Then our updated model is:

$$Vc'(t) = [k + a * \sin(2\pi t) + b * \sin(2\pi t/7)]r + P - c(t)r \tag{2}$$

Questions

Your report should answer the following questions and address the following issues:

1. Classify (1). Interpret (in words) each of the following terms: $kr, Vc'(t), P, c(t)r$ making sure to include units for each.

2. Let $c'(t) = f(c) = (kr + P - c(t)r)/V$. Determine the *singular solution* of (1). What is the physical significance of this singular solution? What does it tell us?

3. Find the general solution of (1). Using any graphing tool available, plot on one graph the concentration levels for varying initial conditions (let $c(0) = 0, 10, 20, ..., 60$) over a period of 180 days. Also, plot the slope field and explain the relationship between the two plots.

4. Find the analytic solution to (2) with the initial concentration $c(0) = 0$. Does the solution have a limit as $t \to \infty$? If so, what is it?

5. Let $a = 20$ and $b = 50$. Plot the concentration over a period of 180 days.

6. Now set $a = 0$ and $b = 50$. What does setting $a = 0$ mean in words? Plot the concentration over a period of 180 days.

7. Now set $a = 20$ and $b = 0$. Plot the concentration over a period of 180 days.

8. Which has a greater effect on the distributed concentration of arsenic in the lake, weekly fluctuations or daily fluctuations? Support your answer.

9. The current standard of 10μ g/liter was set by the EPA in 2006. We will call this the maximum contaminant level (MCL). It is crucial that the company lower its level of arsenic pollution so that the concentration of arsenic in the lake water is within the levels mandated by law. Rather ausipiciously, you have been given two different ways of ensuring compliance with the current policy:

    1. One of your brilliant CU chemical engineers has come up with a process whereby the factory's wastewater is blended with a chemical binder which renders the arsenic inert. Then, this sludge can be safely disposed. The net effect on the model is to reduce the value of k. The cost of this process is $\$300,000$ per μ g/liter reduction (process)

2. One of your ambitious CU civil engineers has discovered an underground spring that does not currently feed into the lake. The spring flows at a rate of 10^{11} liters per day. By diverting some of this clean flow into the lake, the arsenic level in the lake would be diluted. Assume that the flow rate out of the lake would then increase at the same rate, so that the volume of water in the lake remains constant. The estimated cost of diverting this water into the lake is $\$300,000$ per 10^9 liters per day.

First, determine the cost of each of the two processes to lower the arsenic levels to within accepted levels. Then, choose one of these options, and explain briefly which method you would recommend and why.

10. Demonstrate, by the use of plots (choosing an appropriate time scale), that regardless of which option you chose above, that the factory is now in compliance with the EPA standard (you need to give two plots here, one for each option above).

11. Discuss briefly the following issues:

    - What are the weaknesses in the model(2) used?
    - What are additional effects that the pollution model should take into acount?
    - How can the model be improved?

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