APPM 2360 **EXAM 3** **17 November, 1993**

On the front of your bluebook, write your name and your instructor's name.

There are SIX questions. Answer all parts of all 6 questions. Show **all** your work in your bluebook and box in your answers. **Calculators are not permitted.**

1. (16 points)

   (a) Given $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{pmatrix}$

   Calculate $A \cdot B \cdot C$ and $C \cdot B$, if possible.

   (b) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 1 & 2 \\ 3 & 3 \end{pmatrix}$, find

   $C = \begin{pmatrix} p & q \\ r & s \\ t & u \end{pmatrix}$ such that $2A - B + C = 0$.

2. (16 points) Given the following coupled system

$$(*) \begin{cases} \dfrac{d^2u_1}{dt^2} + 2\dfrac{du_1}{dt} + u_2 = 0 \\ \\ \dfrac{d^2u_2}{dt^2} - 3\dfrac{du_2}{dt} - u_1 = 0 \end{cases}$$

   (a) Put $x_1 = u_1,\ x_2 = u_2,\ x_3 = u_1',\ x_4 = u_2'$ where $' = \dfrac{d}{dt}$ and convert $(*)$ to a first-order system for x_1, x_2, x_3, x_4.

   (b) Put in vector form $x' = Ax,\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$.

3. (18 points) Given

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ -1 & -1 & 0 & 1 \\ 1 & 3 & -2 & 7 \\ -1 & 0 & -1 & 5 \end{pmatrix}$$

(a) what is the rank of A?

(b) solve $Ax = 0$, writing solution in vector form.

(c) $\det A =$

4. (16 points) Given

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

(a) find A^{-1}

(b) show that $A^{-1}A = I$

5. (18 points) Given

$$x' = Ax, \quad A = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$$

(a) find eigenvalues and associated eigenvectors for A

(b) find two nontrivial solutions $\bar{x}_1(t)$ and $\bar{x}_2(t)$ of $x' = Ax$.

(c) show that $\bar{x}_1(t)$ and $\bar{x}_2(t)$ are linearly independent.

(d) find the general solution.

6. (16 points) Given $v_1 = (1, 2, 1, 1)$, $v_2 = (1, 5, 0, 3)$, $v_3 = (1, 8, -1, 5)$

(a) determine if they are linearly dependent or independent

(b) If they are linearly dependent, find a linear relation between them.

(c) Show that $(1, 5, 0, 3)$ is or is not in the span of v_1 and v_3.