

On the front of your bluebook, write your name  
and your instructor's name.

There are FIVE questions. Answer all parts of all 5 questions. Show **all** your work in your bluebook and box in your answers. **Calculators are not permitted.**

1. (20 points) Given the following third-order equation

$$(*) \quad y''' - 3y'' + 3y' - y = \sin t$$

(a) Set  $x_1 = y$ ,  $x_2 = y'$ ,  $x_3 = y''$  to convert (\*) to a first-order system.

(b) Put this system in vector form  $x' + Ax = b(t)$  where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad b(t) = \begin{pmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{pmatrix}$$

(c)  $A = \underline{\hspace{2cm}}?$ ,  $b(t) = \underline{\hspace{2cm}}?$

2. (20 points) Let  $A = \begin{pmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 6 & 0 \\ 8 & 5 & 13 & -2 \end{pmatrix}$

(a) What is rank of  $A$ ?

(b) Solve  $Ax = 0$ .

(c) Write the solution as a linear combination of vectors.

(d) Find all solutions (if any exist) of  $Ax = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ .

3. (20 points) Given  $A = \begin{pmatrix} 2 & 4 \\ -2 & -2 \end{pmatrix}$

- (a) find the characteristic polynomial of  $A$ .
- (b) find the eigenvalues and the associated eigenvectors.

4. (20 points) Given

$$x'_1 = 3x_1 + 6x_2$$

$$x'_2 = -x_1 - 2x_2$$

- (a) write in form  $x' = Ax$ ,  $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ .
- (b) find two linearly independent solutions of  $x' = Ax$ .
- (c) find all solutions of  $x' = Ax$ .

5. (20 points) Given  $A = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{pmatrix}$

- (a) calculate  $A^{-1}$ .
- (b) verify your calculated  $A^{-1}$  is correct.
- (c) calculate  $\det A$ .