Final Exam , Fall 1995 Solutions

1) (a) $y(x) = [C + \cos(x)]^{-1}$ (b) $y(x) = \sec(x)$

2) (a) $y(x) = e^{x^2}(C + x)$ (b) $y(x) = e^{x^2}(x - 1)$

3) (a) $y(x) = c_1e^{4x} + c_2e^{-x}$ (b) $Y = -4x + 3$
(c) Simply add the above two solutions.

4) (a) $\lambda^2(\lambda-2)(\lambda-4)$ (b) $\lambda=0,2,4$ (0 is multiplicity 2)
(c) No. Det(A) = 0.
(d) The eigenvectors for $\lambda = 0$ are $\xi^{(1)}= (0,1,0,-2)$ and $\xi^{(2)} = (-1,0,1,0)$

5) (a) T (b) T (c) F (d) F (e) T

6) (a) r=1 multiplicity 2
(b) $\xi = (1,-2)$, $\eta = (0,1) + c(1,-2)$, so $x(t) = \xi e^t + (\xi t+\eta)e^t$
(c) $\Psi(t) = \begin{pmatrix} e^t & te^t \\ -2e^t & (1-2t)e^t \end{pmatrix}$

(7) (a) $x_1' = x_2$, $x_2' = x_3$, $x_3' = x_4$,
$x_4' = -x_4 + 5x_3 + x_2$
(b) $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 5 & -1 \end{pmatrix}$
(c) $r^4+r^3-5r^2-r = 0$

8 (a) $\Psi = \begin{pmatrix} e^{2t} & e^{-t} & 0 \\ e^{2t} & 0 & e^{-t} \\ e^{2t} & e^{-t} & e^{-t} \end{pmatrix}$ (b) $\Psi^{-1}(0) = \begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

(c) $e^{tA} = \Psi(t)\Psi^{-1}(0) = \begin{pmatrix} e^{2t} & e^{2t}-e^{-t} & e^{-t}-e^{2t} \\ e^{2t}-e^{-t} & e^{2t} & e^{-t}-e^{2t} \\ e^{2t}-e^{-t} & e^{2t}-e^{-t} & 2e^{-t}-e^{2t} \end{pmatrix}$

(d) $\left(e^{tA}\right)^{-1} = e^{-tA}$, so rewrite above with $t \to -t$

9 (a) $\Phi(t) = \begin{pmatrix} e^t - 3e^t + 3e^{2t} \\ 0 \qquad e^{2t} \end{pmatrix}$ (b) $X(t) = \begin{pmatrix} -3 \\ -1 \end{pmatrix} e^{3t}$

(c) $x(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} -3 \\ -1 \end{pmatrix} e^{3t}$

10 (a) $\lambda = \pm 2i$. Type is Center. Rotation is CCW. Stable
(b) $\lambda = -1\pm 4i$. Type is Spiral. Rotation is CCW. Asymptotically Stable.
(c) $\lambda = 1\pm 2i$. Type is Spiral. Rotation is CW. Unstable
(d) $\lambda = -2,1$. Type is Saddle, Eigenvectors (1,2) and (1,3). Unstable.