Final Exam

APPM 2360, Fall 1996
Dec. 14, 1996

ON THE FRONT OF YOUR BLUEBOOK write (1) your name, student #, (2) lecture section and instructors name (020 for Meiss, 030 for Li, and 040 for Curry), and (3) Put a grading grid on your exam.

There are TEN questions. You must work all of the problems. Show ALL of your work in your bluebook and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Start each problem on the top of a new page. You may have no papers or books or devices of any sort (calculators, nuclear bombs, computers, etc.) on your desk.

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1) (20 pts) Let

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad Q = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

a) For each matrix, either compute its inverse or else show that it is singular.
b) Compute the eigenvalues $\lambda_1 \le \lambda_2 \le \lambda_3$ of P.
c) Compute the eigenvector of P corresponding to the largest eigenvalue, λ_3, of P.

2)(20 pts) Show that the following sets of vectors are linearly dependent or linearly independent

a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 8 \\ 0 \end{pmatrix} \right\}$ b) $\left\{ \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \end{pmatrix} \right\}$

c) $\left\{ \begin{pmatrix} 1 \\ 2e^{2t} \end{pmatrix}, \begin{pmatrix} e^{t} \\ 2e^{3t} \end{pmatrix} \right\}$ d) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} \right\}$

3a) (20 pts) A 10 lb Christmas roast beast, initially at 50° F is placed in a 375° F oven at 11:00 in the morning. 75 minutes later the temperature of the roast is 125° F. When will the temperature be 150° F. (*Hint*: Newton's law of cooling states that the rate of change of temperature is proportional to the difference between the ambient temperature and the current temperature).

4) (20 pts) a) Determine all critical points of the equations

$$\frac{dx}{dt} = y - 1, \quad \frac{dy}{dt} = y^2 - x^2$$

b) For each of the critical points in (a), determine whether they are stable, asymptotically stable, or unstable.

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5) (20 pts) Solve the following initial value problems

a) $y'' - 5y' - 6y = 0, \quad y(0) = 0, y'(0) = 1$

b) $y' + y = 2 \quad y(1) = 0$

6) (20 pts) Find the general solution of the following differential equations

a) $2ty' - 3y = 9t^2$

b) $y'' - 4y' + 4y = 0$

7) (20 pts) Indicate whether the following statements are always TRUE or otherwise FALSE.

a) The equation $\frac{dy}{dt} = y\sec(t)$, where y(1)=37 has a unique solution in the interval $-\pi < t < \pi$.

b) If there are three vectors $\{x^{(1)}, x^{(2)}, x^{(3)}\}$ and $x^{(1)} = 2x^{(2)} - x^{(3)}$ then they are linearly independent.

c) If $\boldsymbol{A}$ is a 2×3 matrix and $\boldsymbol{B}$ is 3×2 matrix then $\boldsymbol{AB}$ is a 2×2 matrix.

d) If $y_1(t)$ and $y_2(t)$ are two independent solutions of the equation $y'' + 3y' + sin(t)\, y = 0$, then their Wronskian is 0.

e) If $\boldsymbol{A}$ is a real matrix and has a complex eigenvalue $r_1 = \lambda + i\mu$ then it also has the eigenvalue $r_2 = -\lambda + i\mu$.

8)(20 pts) Consider the equation

$$y'' + y = 3\sin t$$

a) Find the general real solution of the associated homogeneous equation.

b) Find the general real solution of the original nonhomogenous equation.

9)(20 pts) A damped spring undergoing a constant force is modeled by the equation

$$4y'' + 8y' + 5y = 1$$

a) Convert this equation to a system of first order differential equations.

b) Find the critical point of this system (x_1^o, x_2^o)

c) Sketch the phase plane portrait for the solutions of this system.

d) Is the critical point stable, asymptotically stable or unstable?

10) (20 pts) Determine the most general form of the real solutions of the following linear differential equations. Determine a fundamental matrix solution $\Psi(t)$ and the associated matrix exponential, $\Phi(t) = e^{At}$.

a) $x' = \begin{pmatrix} 0 & -4 \\ 1 & 0 \end{pmatrix} x$

b) $\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$