

APPM 2360 - Exam 1

February 16, 2000 , 7 - 8.30 pm

On the front of your blue book, write your name. Write also the names of your lecturer (or lecture session number) and your TA (or recitation section number).

There are FIVE problems (with subparts a, b, ...). YOU MUST WORK ALL FIVE PROBLEMS. Each full problem is worth 20 points. With the exception of problem 4 (which is 'multiple choice', and requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

1. Consider the ODE $y' = y + 1$.
 - a. Solve this equation both by using integrating factor and by separation of variables (make sure you get the same answer with both methods).
 - b. Determine if the equation has any *nullclines* and/or *equilibrium solutions*. If it does, give the equations for these.

2. Consider the ODE
$$\begin{cases} y' = e^{-y} \\ y(0) = 1 \end{cases}$$
 - a. Find the analytic solution to this initial value problem.
 - b. Determine (from the analytic solution of the ODE) the largest t -interval (from negative to positive values) over which the solution exists.
 - c. For what rectangles in the (t, y) -plane can we apply the *Existence and Uniqueness theorem*? (in the text book described as Existence and Uniqueness theorem with Extension Principle).
 - d. Does your answer on part c contradict the one on part b? Explain!

3. Consider the ODE $y' = \sqrt{1 - y^2}$ with initial condition $y(0) = 0$.
 - a. Is $y(t) = \sin t$ a solution?
 - b. Make a rough sketch of the direction field for the ODE. Indicate where the slope is positive, negative, zero, and undefined.

c. Determine the value of $y(\pi)$.

Please turn over \Rightarrow

4. A 50 gallon tank initially holds 20 gallons of water, containing 5 lbs of salt. Starting at $t = 0$, a solution containing 3 lbs of salt per gallon flows into the tank. The rate of inflow at time t is $(5 + \sin t)$ gal/min. The outflow rate is 5 gal/min. The tank is well stirred at all times.

(This problem is a 'multiple choice' problem. You need only to select your preferred answers.)

a. Let $y(t)$ be the amount of salt, measured in pounds, in the tank at time t . Specify the initial condition: $y(0) =$

- (A) 3
- (B) 5
- (C) 20
- (D) 50

b. Let $V(t)$ be the volume of liquid in the tank at time t . $V(t) =$

- (A) 20
- (B) $20 + \sin t$
- (C) $20 + \cos t$
- (D) $21 - \cos t$

c. The ODE which correctly models $y(t)$ is

- (A) $y'(t) = (5 + \sin t) - 5$
- (B) $y'(t) = 3(5 + \sin t) - (y(t)/20) * 5$
- (C) $y'(t) = 3(5 + \sin t) - (y(t)/V(t)) * 5$
- (D) $y'(t) = (5 + \sin t) - y(t)/20$

5. Although we in general cannot solve analytically a second order ODE

$$y'' = F(t, y, y'),$$

there are some special cases in which we at least can reduce the equation to first order, making it more amenable to analytical solution. With this observation in mind, solve

a.
$$y'' = -\frac{2}{1-y} (y')^2$$

A *Bernoulli equation* is an ODE of the form $\frac{dy}{dx} + p(x)y = q(x)y^n$. If $n = 0$ or $n = 1$, the equation is first order linear (and can therefore be solved by integrating factor). For all other values of n , it can be solved via a change of dependent variable $u(x) = y(x)^{1-n}$.

b. On the Lab 1, you solved $y' = ay - cy^2$ by separation of variables. Solve again this same ODE, but now making use of the fact that it is a special case of a Bernoulli equation.