

**APPM 2360 - Exam 2****March 15 , 2000 , 7 - 8.30 pm**

On the front of your blue book, write your name. Write also the names of your lecturer (or lecture session number) and your TA (or recitation section number).

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There are FIVE problems (with subparts a, b, ...). YOU MUST WORK ALL FIVE PROBLEMS. Each full problem is worth 20 points. With the exception of problem 3 (which is 'multiple choice', and requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

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1. Given the ODE  $y'' + 4y' + 4y = 4t$ 
  - a. Find the general real valued solution of the associated undriven ODE.
  - b. Find a particular solution to the driven ODE.
  - c. Find the solution for the initial values  $y(0) = 0, y'(0) = 1$ .
  
2. Consider the following ODEs:
  - i.  $y'' - 25y = 0$
  - ii.  $y'' + 25y' = 0$
  - iii.  $y'' + 25y = 0$
  - a. For each of the equations above, find the general solution.
  - b. Find the general solution to  $y'' - 25y = e^{5t}$ .
  - c. For  $y'' + 25y = \sin 5t$ , will the solution in the long run grow, decay, or have constant amplitude? Explain your answer.

Hint: You can answer the question c without solving the equation.

Please turn over  $\Rightarrow$

3. Suppose  $w_1(t)$  and  $w_2(t)$  are two solutions to the undriven ODE  $y'' + ay' + by = 0$  and  $z_1(t)$  and  $z_2(t)$  are two solutions to the driven ODE  $y'' + ay' + by = \cos t$ .

State whether each of the following statements are TRUE or FALSE (on this problem, you do not need to show any of your work):

- a.  $z_1 + z_2$  is a solution of  $y'' + ay' + by = \cos t$ .
  - b.  $z_1 + w_1 + w_2$  is a solution of  $y'' + ay' + by = \cos t$ .
  - c.  $(D^2 + aD + b)[w_1 + 3w_2] = 0$ .
  - d. The only solution to the ODE  $y'' + ay' + by = 0$ , with initial condition  $y(0) = 0$  is the trivial solution  $y(t) = 0$ .
4. Consider the following second order ODE describing an undamped mass-spring system (suspended vertically):  $my'' + ky = -mg$
- a. Find the equilibrium solution(s) of this ODE.
  - b. Now suppose the spring-mass is driven by a force  $f(t) = \cos 2t$ . Also, let  $m = 1$ ,  $k = 1$ . That is, the spring-mass position satisfies the modified ODE  $y'' + y = -g + \cos 2t$ . Find a particular solution to this ODE.
5. Given the ODE  $y' = (y - c)(y^2 + c)$ .
- a. What are the equilibrium solutions for  $c > 0$ ,  $c < 0$ , and  $c = 0$ ?
  - b. Sketch the bifurcation diagram. Label equilibrium values as stable (attractors) or unstable (repellers). Label your axes.