

APPM 2360 - Exam 3

April 19 , 2000 , 7 - 8.30 pm

On the front of your blue book, write your name. Write also the names of your lecturer (or lecture session number) and your TA (or recitation section number).

There are FIVE problems (with subparts a, b, ...). YOU MUST WORK ALL FIVE PROBLEMS. Each full problem is worth 20 points. With the exceptions of part a of problems 1 a and 5 (which are 'multiple choice', and requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

1. a. In the following three systems of ODEs, the functions $x(t)$ and $y(t)$ represent the populations of two interacting species.

$$\begin{array}{ll} 1. & x' = (-2 - x + y) x & y' = (4 - x + .5 y) y \\ 2. & x' = (2 - x + 4 y) x & y' = (1 + 3 x - y) y \\ 3. & x' = x - x^2 - x y & y' = 1 - y^2 - x y \end{array}$$

Select from the list below of biological relationships the one which best describes the relationship between species x and species y .

- A. x is the predator and y is the prey
- B. y is the predator and x is the prey
- C. Species x and y cooperate
- D. Species x and y are in competition
- E. None of the above.

Present your answers in a table like the one below. No explanation of your work is required.

Equation	1	2	3
Relationship			

b. Plot the nullclines and equilibrium solutions for the system of equations below. Clearly label your graph.

$$\begin{array}{l} x' = (-2 - x + y) x \\ y' = (4 - x + .5 y) y \end{array}$$

Please turn over \Rightarrow

2. For each of the following three ODEs

- i. Write the equation as an equivalent first order system.
- ii. If the system can be written in the form $x' = A x + F$, find the matrix A and the vector F (both independent of x). If it can not, explain why not.

a. $x''' + 4 x' + 7 x = \cos t$

b. $x'' + t^2 x + e^t x = 4 x$

c. $x'' + x x' = 0$

3. Find all solutions to the following systems of equations

a.
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 1 \\ x_1 + 2 x_2 + 3 x_3 + 4 x_4 &= 5 \\ 2 x_1 + x_2 + 4 x_3 + 3 x_4 &= 2 \end{aligned}$$

b.
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 1 \\ x_1 + 2 x_2 + 3 x_3 + 4 x_4 &= 5 \\ 2 x_1 + x_2 + 4 x_3 + 3 x_4 &= 6 \end{aligned}$$

4. Find the eigenvalues and associated eigenvectors of the following matrices

a.
$$\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

5. Determine whether the following statements are TRUE or FALSE. You do not need to show your work. Please write out TRUE or FALSE (not T or F).

a.
$$\det \left(2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 2.$$

b. If both v_1 and v_2 are eigenvectors of A with eigenvalue 3, then $v_1 + v_2$ is also an eigenvector of A with eigenvalue 3.

c. The vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$.

d. If $\det A$ is not equal to zero, then $A^T x = b$ has a unique solution.

e. The three vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ form a basis for \mathbb{R}^3 .