

On the front of your blue book, write your name. Write also the names of your lecturer (or lecture session number) and your TA (or recitation section number).

There are FIVE problems (with subparts a, b, ...). YOU MUST WORK ALL FIVE PROBLEMS. Each full problem is worth 40 points. With the exception of problem 5 (which is 'multiple choice', and requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

1. Solve the following ODEs. Give the general solution unless initial conditions are specified.

a. $y' - \frac{3y}{2t} = \frac{9}{2}t^2$.

b. $t^2 y' + y^2 = 0$.

c. $y'' = 2y^3, \quad y(0) = 1, \quad y'(0) = 1.$

2. a. Sketch the bifurcation diagram for $x' = x^3 + 2x^2 + cx$. Label the axes. Indicate which equilibrium points are stable (attractors) and which are unstable (repellers).

b. Suppose $c = -3$ (that is, $x' = x^3 + 2x^2 - 3x$). Sketch representative solution curves $x(t)$ for this ODE. Identify equilibrium solutions on the graph.

c. Consider the following system:

$$\begin{aligned} x' &= x^3 + 2x^2 - 3x \\ y' &= xy \end{aligned}$$

Sketch the nullclines of this system (identify which are x -nullclines and which are y -nullclines). Identify all equilibrium points.

3. Solve the system of differential equations

$$\begin{cases} x' = 3x + y + 4z \\ y' = x + 3y - 2z \\ z' = 2z \end{cases}$$

with initial conditions

$$x(0) = 2, \quad y(0) = -1, \quad z(0) = -1.$$

Please turn over \Rightarrow

