APPM 2360 - Exam 1 **September 27, 2000 , 7 - 8.30 pm**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and your TA (or recitation section number). Draw also a grading grid.

There are FIVE problems (with subparts a, b, ...). YOU MUST WORK ALL FIVE PROBLEMS. Each full problem is worth 20 points. Start each problem on a new page. With the exception of problem 5 (which requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

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1. Consider the differential equation

$$3y' - ty - t = 0.$$

a. Find the general solution.

b. Find the solution which satisfies the initial condition $y(1) = 0$.

2. Initially a tank contains 1,000 gallons of water. Fluid enters the tank from two pipes. One pipe pumps in pure water at 3 gallons per minute while the other pumps in salt water with a concentration of 1 pound of salt per gallon at the rate of 5 gallons per minute. A well mixed solution leaves the tank at the rate of 8 gallons per minute.

a. Derive the initial value problem for $y(t)$, the number of pounds of salt in the tank at time t.

b. Solve the initial value problem.

3. Solve the initial value problem

$$y'' + y = 0,\ y(0) = 0,\ y'(0) = 1.$$

Hint: You may find use for the relation $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C.$

4. Consider the following differential equation

$$y' = y^2 + y - 2.$$

a. Determine the equilibrium solutions to the ODE.

b. By means of the Existence / Uniqueness theorem (and it's extension), demonstrate that the solution satisfying $y(0) = -1$ exists and remains unique for the complete interval $-\infty < t < \infty$.

c. Solve the ODE, again with the initial condition $y(0) = -1$.

Please turn over $\Longrightarrow$

5. For each slope field (a) - (d) shown below, identify which of the differential equations (1) - (6) it depicts. (Note that two of the differential equations are not illustrated with a slope field).

(1) $\frac{dy}{dt} = y\cos(\pi t)$

(2) $\frac{dy}{dt} = t - y^2$

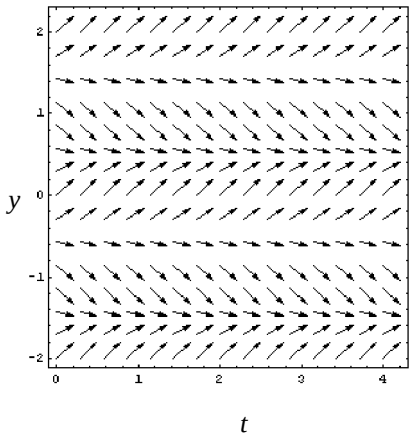
(3) $\frac{dy}{dt} = 1 - y^2$

(4) $\frac{dy}{dt} = t\cos(\pi y)$

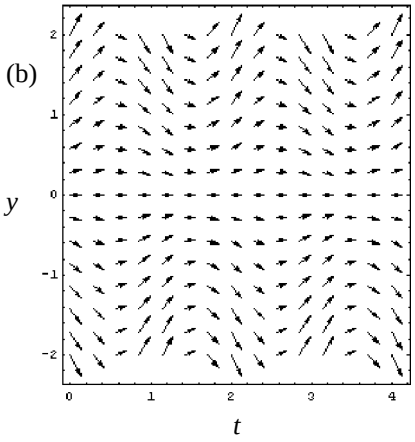
(5) $\frac{dy}{dt} = \cos(\pi y)$

(6) $\frac{dy}{dt} = y^2 - 1$

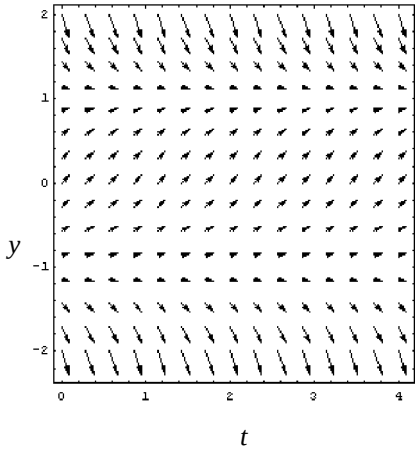
(a)



(b)



(c)



(d)