APPM 2360 - Exam 2 **November 1, 2000 , 7 - 8.30 pm**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and your TA (or recitation section number). Draw also a grading grid.

There are FOUR problems (with subparts a, b, ...). YOU MUST WORK ALL FOUR PROBLEMS. Each full problem is worth 25 points. Start each problem on a new page. With the exception of problem 4 (which requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

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1. Consider the following one-parameter family of differential equations

$$y' = y(1 - a^2 - y^2)\ .$$

a. Find all equilibrium points. Describe how they depend on the parameter a.

b. Draw the bifurcation diagram. Indicate with arrows where solutions are increasing or decreasing. Label all values of the parameter a for which a bifurcation point occurs. Mark which branches are stable (attracting) and unstable (repelling).

2. Find the general solution to the ODE $y'' - 5y' + 6y = f(t)$ where

a. $f(t) = 1$

b. $f(t) = 2\,e^{2t}$

c. $f(t) = 1 + 2\cos 2t$

3. Consider a damped oscillator satisfying

$$\theta'' + 3\theta' + 2\theta = 0$$

with initial conditions

$\theta(0) = \theta_0$ (where θ_0 is unknown),
$\theta'(0) = 0$.

a. Determine the time it takes until $\theta(t)$ has decreased to half of its original value (i.e. to $\theta(t) = \frac{1}{2}\theta_0$).

b. Would it take twice as long (as in part a) for $\theta(t)$ to decrease to a quarter of its original value? Explain your answer.

Hint: You may find it helpful to substitute $x = e^{-t}$ at some point in the problem.

Please turn over ⇒

4. Multiple choice - no explanations are needed for your answers. Your answers should be in the form of a table as shown below and in which you place one (and only one) cross in each table row:

	i	ii	iii	iv
4 a				
b				
c				
d				
e				

a. The minimal degree of a linear constant coefficient undriven ODE which has as solutions $y = t$ and $y = t\,e^{-t}$ is

i) 2
ii) 3
iii) 4
iv) There is no such ODE

b. The four numerical schemes below for solving $y' = f(t, y)$ (with h denoting the spacing in time) are all possible to use. Identify which one of the following is known as "Euler's method"

i) $y_{n+1} = y_{n-1} + 2h\,f(t_n, y_n)$
ii) $y_{n+1} = y_n + \frac{1}{2}h\,[3\,f(t_n, y_n) - f(t_{n-1}, y_{n-1})]$
iii) $y_{n+1} = y_n + h\,f(t_n, y_n)$
iv) $y_{n+1} = y_n + \frac{1}{2}\,h[f(t_{n+1}, y_{n+1}) + f(t_n, y_n)]$

c. One of the following four ODEs has unbounded solutions as $t \to \infty$. Identify which one it is:

i) $y'' + 4y' + 3y = 2 + 6e^{-t}$
ii) $y'' + 4y' + 4y = 0$
iii) $y'' + 9y = e^{-t} + \cos 3t$
iv) $y'' + y' = 100 \sin 100t$

d. Given that $y(t)$ solves the ODE $P(D)[y] = i\,e^{it}$, what ODE does $z(t) = Re(y(t))$ satisfy

i) $P(D)[z] = \cos t$
ii) $P(D)[z] = \sin t$
iii) $P(D)[z] = -\sin t$
iv) $P(D)[z] = e^{-t}$

e. The equation $y'' + a(t)\,y = f(t)$ has $y_1 = t^2$ and $y_2 = t^4$ as solutions (on the interval (0,1)). The function $a(t)$ is then

i) $a(t) = \frac{2(1-6t^2)}{t^2(t^2-1)}$
ii) $a(t) = \frac{1}{t^2}$
iii) $a(t) = t^2 - t^4$
iv) $a(t) = \frac{1}{1-t^2}$