APPM 2360 - Exam 3 November 29, 2000 , 7 - 8.30 pm

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and your TA (or recitation section number). Draw also a grading grid.

There are FOUR problems (with subparts a, b, ...). YOU MUST WORK ALL FOUR PROBLEMS. Each full problem is worth 25 points. Start each problem on a new page. With the exception of problem 4 (which requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

---

1. Determine the values of a for which the linear system

$$\begin{aligned} x_1 - x_2 + a x_3 &= 0 \\ 3x_1 - 3x_2 + x_3 &= 0 \\ a x_1 - x_2 + 2x_3 &= 0 \end{aligned}$$

has non-trivial solutions (i.e. not all components of the solution vector zero).

2. Given the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 2 & 1 & -1 \\ 2 & 6 & 1 \end{bmatrix}$.

   a. Determine all the eigenvalues of A.

   b. Verify that your result in part a is consistent with the formula $tr(A) = \Sigma a_{ii} = \Sigma \lambda_i$.

   c. Find the eigenvectors corresponding to each eigenvalue of A. If an eigenspace is deficient, find also the generalized eigenvector(s).

3. Solve the system of ODEs

$$\begin{aligned} x_1' &= 3x_1 + 8x_2 \\ x_2' &= -x_1 - 3x_2 \end{aligned}$$

with initial conditions $x_1(0) = 6$, $x_2(0) = -2$.

4. Multiple choice - no explanations are needed for your answers. Your answers should be in the form of a table as shown at the top of the next page. You should place one (and only one) cross in each table row to indicate which choice is correct:

Please turn over ⇒

	i	ii	iii	iv
4 a				
b				
c				
d				
e				

a. The matrix $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ has the eigenvalues

i) $\lambda_1 = a + b$, $\lambda_2 = a - b$
ii) $\lambda_1 = a$, $\lambda_2 = b$
iii $\lambda_1 = \lambda_2 = a + b$
iv) $\lambda_1 = \lambda_2 = a - b$

b. Consider the ODE system $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ where A is a 3×3 matrix with eigenvalues -3, -2 and 0. Then

i) There exists a solution that goes to infinity as $t \to \infty$
ii) All solutions go to infinity as $t \to \infty$
iii) All solutions are periodic
iv) No solution goes to infinity as $t \to \infty$

c. The ODE system $\begin{cases} x' = 6x - x^2 + 3xy \\ y' = y + 2xy - 2y^2 \end{cases}$ describes the time evolution of the populations of two species. The dynamics can be described as

i) Competition
ii) Predator - prey
iii) Cooperation
iv) No interaction

d. If A and B are square matrices of the same size, then

i) $A\,B = B\,A$
ii) $\det(A^T) = \det(A^{-1})$
iii) $\det(A\,B) = \det(B\,A)$
iv) $\det(A + B) = \det(A) + \det(B)$

e. Given the constant coefficient driven system of ODEs $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} t^2 \\ 2t \end{bmatrix}$. Then x_1 satisfies the following second order ODE

i) $x_1'' - 4x_1' + 4x_1 = -t^2 + 4t$
ii) $x_1'' + 3x_1' + 3x_1 = 0$
iii) $x_1'' - 4x_1' + 4x_1 = 3t^2$
iv) None of the above