APPM 2360 - Final **December 20, 2000 , 4.30 - 7.00 pm**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and your TA (or recitation section number). Draw also a grading grid.

There are SIX problems (with subparts a, b, ...). YOU MUST WORK ALL SIX PROBLEMS. The first five problems are worth 35 points each; the last (multiple choice one) is worth 25 points. Start each problem on a new page. With the exception of problem 6 (which requires only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

---

1. Find the general solution $y(t)$ to the following ODEs:

   a. $y' = e^{t+y}$

   b. $y' - y = 5\,e^{t}$ by integrating factor

2. The autonomous system of equations

$$x' = 3x - 3x^2 + xy$$
$$y' = -2y + 4xy$$

describes how the populations evolve for two interacting species.

   a. Sketch in a population quadrant ($x \geq 0$, $y \geq 0$) the orbits of this system. In particular, mark nullclines and equilibrium points. Add a representative number of trajectories, and put arrows on them.

   b. Explain what happens to the population for large times.

3. Draw the bifurcation diagram for

$$y' = (y^2 + c^2 - 1)\,(y + c - 1)$$

Label your axes clearly and mark your equilibrium lines solid (if stable; attractor) or dashed (if unstable; repeller). Mark clearly the bifurcation points. Tell how many there are.

4. a. For $y(t) = h(t)\,e^{t}$ to be a solution to the ODE $P(D)[y] = (D-1)(D+2)[y] = f(t)\,e^{t}$, what ODE must $h(t)$ satisfy?

   b. Solve the following second order ODE:

$$y'' = \frac{8(y-1)}{(y')^2}\ ,\ \ y(0) = 1,\quad y'(0) = 0.$$

5. Given the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

Please turn over ⇒

a. Find the eigenvalues, eigenvectors and (if necessary) generalized eigenvectors to A.

b. Write down the general solution to the ODE system $x' = Ax$.

6. Multiple choice - no explanations are needed for your answers. Your answers should be in the form of a table as shown to the right. You should place one (and only one) cross in each table row to indicate which choice is correct.

	i	ii	iii	iv
6 a				
b				
c				
d				
e				

a. What is the lowest degree possible for a constant coefficient linear ODE with real coefficients that has $\sin t + \cos t + t^2$ among its solutions?

i) 3 ii) 5 iii) 6 iv) 7

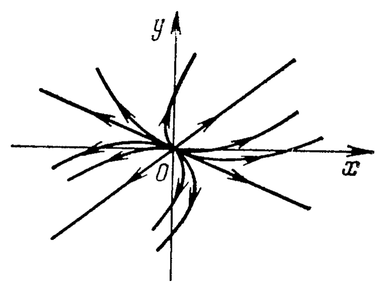
b. The figure to the right shows the phase portrait of which one of the following four ODE systems

i) $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} -2 & -1 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

ii) $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

iii) $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 8 & 8 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

iv) $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$



c. One of the following ODEs features a resonance. Identify which one it is

i) $y'' + y = \cos t$
ii) $y'' - y = \sin t$
iii) $y'' + y = e^t$
iv) $y'' + 25y' = 5e^{it}$

d. Applying Euler's method with step size $h = \frac{1}{2}$ to numerically solve $y' = ty$, $y(0)=1$ produces for $y(1)$ the answer

i) 1 ii) 5/4 iii) 2 iv) 7/4

e. Given that the relations $P(D)y_1 = \sin t$ and $P(D)y_2 = e^t$ hold, then the relation

$$P(D)\,y = 3\sin t + e^t$$

holds if

i) $y(t) = 3\sin t + e^t$
ii) $y(t) = 3\,y_1(t) + y_2(t)$
iii) $y(t) = y_1(t) + y_2(t)$
iv) None of the above