APPM 2360 - Exam 3 **April 25 , 2001 , 7 - 8.30 pm**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and your TA (or recitation section number). Draw also a grading grid.

There are FOUR problems (some with subparts a, b, ...). YOU MUST WORK ALL FOUR PROBLEMS. Each full problem is worth 25 points. Start each problem on a new page. With the exception of problem 4 (which require only the answers), show all your work in your bluebook. Box all your answers. Calculators, books or any notes are NOT permitted. No 'crib sheets' are allowed.

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1. Determine the values of the constant a for which the linear system $\begin{bmatrix} a & 1 & \frac{3}{2} \\ -2 & a & 0 \\ 0 & a & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has non-trivial solutions (not $x_1 = x_2 = x_3 = 0$)?

2. Determine the general solution to the following two systems of ODEs (give the answers in real form):

   a. $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 2 & 5 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$

   b. $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$

   c. For each of the two cases above, describe the equilibrium point at the origin as one of the four choices below:

      i. hyperbolic, source
      ii. -"- , sink
      iii. -"- , saddle
      iv. non-hyperbolic.

3. Consider the following nonlinear system of ODEs:

$$\begin{cases} x' = \sin(x+y) \\ y' = (x-1)^2 + y^2 - 1 \end{cases}.$$

   a. Determine the two equilibrium points of the system.

   b. Choose any one of the equilibrium points and determine the linearized system around this point.

Please turn over ⇒

4. Multiple choice: For each of the five statements below, enter either a "**T**" in the **True** column or an "**F**" in the **False** column in a table that is laid out as follows:

	True	False
a		
b		
c		
d		
e		

a. The vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ are linearly independent.

b. We want to solve $\begin{bmatrix} x \end{bmatrix}' = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x \end{bmatrix} + \begin{bmatrix} 3e^{-t} \\ 2t \\ te^{-t} \end{bmatrix}$ where A has the eigenvalues

$\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = -2$. An appropriate guess for a particular solution for the system would be

$$\begin{bmatrix} \varphi(t) \end{bmatrix} = \begin{bmatrix} (a_1 t + b_1)\, e^{-t} \\ (a_2 t + b_2)\, e^{-t} \\ (a_3 t + b_3)\, e^{-t} \end{bmatrix}.$$

c. The linear system $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$ has more than one solution.

d. $\begin{bmatrix} x(t) \end{bmatrix} = \begin{bmatrix} e^{-t} \\ t^2 \\ t \end{bmatrix}$ solves the ODE system $\begin{bmatrix} x \end{bmatrix}' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1/t & 1 \\ \frac{1}{2}e^{t} & 1/(2t^2) & 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix}$.

e. If $\underline{\varphi}_1$ and $\underline{\varphi}_2$ solve the linear constant coefficient ODE system $\underline{x}' = A\underline{x} + \underline{b}$ (where $\underline{b} \neq \underline{0}$), then the sum $\underline{\varphi}_1 + \underline{\varphi}_2$ also solves $\underline{x}' = A\underline{x} + \underline{b}$.