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**INSTRUCTIONS:**

- Computers, calculators, books are not permitted. A one page handwritten sheet of notes is permitted.
  - Write your name, instructor's name, and recitation number on the front of your bluebook.
  - Work all five problems. Start each problem on a new page.
  - Show your work clearly and box your final answer.
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1. (20 points) Find the general solution to the initial-value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 0, y'(0) = -1.$$

2. (20 points) Consider the following initial-value problem:

$$y'' + y = 6 \sin(2t), \quad y(0) = 0, \quad y'(0) = 0$$

- (a) (5 points) Find the homogeneous solution.
- (b) (5 points) Compute the Wronskian
- (c) (10 points) Use the method of **variation of parameters** to find the general solution of the above system.
3. (20 points) Mark the following as **True** or **False**
- (a) The solution of  $y'' + 4y' + 13y = 0$  with the initial values  $y(0) = 2$  and  $y'(0) = -2$  is real-valued.
- (b) A repelling spiral has two complex eigenvalues with positive real parts.
- (c) The matrix  $\begin{pmatrix} 3 & 0 \\ 2 & -3 \end{pmatrix}$  has two linearly independent eigenvectors.
- (d) If  $\text{Det}(\mathbf{A}) = |\mathbf{A}| = 0$  then  $\mathbf{A}$  has **no** eigenvalues.
- (e) A  $3 \times 3$  matrix with real coefficients can have 3 complex eigenvalues.
- (f) Let  $\mathbf{A}$  be a  $2 \times 2$  matrix with  $\text{Tr}(\mathbf{A}) = -1$  and  $\text{Det}(\mathbf{A}) = |\mathbf{A}| = -3$ . Then the characteristic polynomial associated with the matrix  $\mathbf{A}$  is  $\lambda^2 - \lambda - 3$ .
4. (20 points) Find the solutions to the following initial values problems

- (a) (10 points)

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

- (b) (10 points)

$$\mathbf{x}' = \begin{pmatrix} 2 & 4 \\ -2 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

5. (20 points) Consider the four differential equations  $\mathbf{x}' = \mathbf{M}\mathbf{x}$  with the matrix  $\mathbf{M}$  given by each of the following matrices

$$\mathbf{A} = \begin{pmatrix} -4 & -2 \\ 2 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ -3 & -2 \end{pmatrix}.$$

For each matrix  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\mathbf{D}$ :

- i Classify the type of the motion (repelling or attracting node, repelling or attracting spiral, center or saddle).
- ii Classify the eigenvalues (i.e., are the eigenvalues real or complex? Give the signs of their real parts).
- iii Is the equilibrium  $(0, 0)$  stable or unstable?
- iv If there is a double eigenvalue find both the eigenvector  $\mathbf{v}$  and the generalized eigenvector  $\mathbf{u}$  for the matrix.