
INSTRUCTIONS:

- Calculators, books, notes, and crib sheets are not permitted.
 - Write your name, instructor's name, and recitation number on the front of your bluebook.
 - Work all five problems. Start each problem on a new page.
 - Show your work and clearly identify your final answer.
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1. (20 points) Consider the initial value problem (IVP)

$$ty' = t^2y^2 - y - 1, \quad y(1) = 0, \quad t > 0$$

- a. (6 points) Use the substitution $u = yt$ to construct the the separable IVP

$$u' = u^2 - 1, \quad u(1) = 0.$$

- b. (10 points) Determine the solution $u(t)$ to the equation derived in part (a).

- c. (4 points) Determine the solution $y(t)$ to the original equation.

2. (20 points) Consider the following equation for $y(t)$:

$$y' \cos t = (y + \cos t) \sin t .$$

- a. (2 points) Write the equation in the normal form.

- b. (8 points) Write down the corresponding *homogeneous* equation and find its general solution.

Hint: you may find the following integral useful: $\int \frac{f'(x)}{f(x)} dx = \log[f(x)]$.

- c. (8 points) Find a particular solution of the *nonhomogeneous* equation using the method of

Variation of Parameters.

- d. (2 points) Determine the general solution of the equation.

3. (20 points) Initially, a 100-liter tank contains a salt solution with concentration 0.5kg/liter. A fresher solution with concentration 0.1kg/liter flows into the tank at the rate of 4 liter/min the contents of the tank are kept well stirred, and the mixture flows out at the same rate it flows in.

- a. (7 points) Write down the initial-value ODE problem that describes the amount of salt in the tank.

- b. (7 points) Using the method of integrating factors find the amount of salt in the tank as a function of time.

- c. (4 points) Determine the concentration of salt in the tank at any time.

- d. (2 points) Find the steady-state concentration of salt in the tank.

4. (20 points) Consider the following differential equation

$$y' = -y(1 - \epsilon y^2)$$

- a. (4 points) Classify the equation for $\epsilon = 0$ and $\epsilon = 1$.
 - b. (12 points) Draw the phase lines for $\epsilon < 0$, $\epsilon = 0$, and $\epsilon > 0$.
 - c. (4 points) Describe the effect that ϵ has upon the stability of the equilibrium solutions.
5. (20 points) Answer the following True/False questions:

- a. $y' - 2/y = 1$ is a linear equation.
- b. $y' - 2/y = 1$ is a separable equation.
- c. Picard's theorem of existence and uniqueness guarantees that the solution of the initial-value problem $y' = (y - t)^{1/3}$ and $y(0) = 0$ is *non-unique*.
- d. If y_1 and y_2 are two different solutions of the equation $y'' + t^2 y = 1$, then $y_3 = (y_1 - 2y_2)$ is also a solution of this equation.
- e. $L[y] = 2y' - 1$ is a linear operator.