
INSTRUCTIONS:

- Calculators, books, notes, and crib sheets are not permitted.
- Write your name, instructor's name, and recitation number on the front of your bluebook.
- Work all five problems. Start each problem on a new page.
- Show your work and clearly identify your final answer.

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1. Consider the following first-order system of ODE's

$$\begin{aligned} \dot{x} &= 2x - xy \\ \dot{y} &= -3x + \frac{1}{2}xy \end{aligned}$$

a. Find all the equilibrium points (5 points)

b. Find and sketch the x-nullcline(s). Annotate the nullcline with the appropriate vector directions. (5 points)

c. Find and sketch the y-nullcline(s). Annotate the nullcline with the appropriate vector directions. (5 points)

c. Sketch the complete phase plane by combining your answers from a.–c. In the open regions of the phase plane sketch the vector directions. (5 points)

2. Solve the following the system of equations by Gaussian Elimination or putting it into Row Reduced Echelon Form (20 points).

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \ , \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \ , \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \ , \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \ . \end{aligned}$$

3. Consider the system of equations

$$\begin{aligned} -x - 3y + z &= 0 \ , \\ 9x + 5y + 2z &= 0 \ , \\ 2y - z &= 0 \ . \end{aligned}$$

a. Do these equations have a unique solution? (6 points)

b. Find the span of the set of solutions of these equations. (10 points)

c. Does the set of solutions of these equations form a basis for $\mathbb{R}^3$? (4 points)

4. **a.** Is the set $\{1, t, 2t^2 - 1, 4t^3 - 3t\}$ linearly independent? (7 points)

   b. Is the set $\{t, t(t+1), t(t-1)\}$ linearly independent? (7 points)

   c. Suppose that $f(t)$ is differentiable and positive for all t. Are $f(t)$ and $tf(t)$ linearly independent? (6 points)

5. Answer the following true/false questions

   a. The solutions of the differential equation $y'' + 2y' - 1 = 0$ form a vector space.

   b. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$. Then A and A^T have the same Reduced Row Echelon Form.

   c. If $\text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\} = \mathbb{R}^n$ then the vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ are linearly independent.

   d. The basis of a vector space is not unique.

   e. If a solution to the system of equations $\mathbf{Ax} = \mathbf{b}$ exists, this implies that the inverse of $\mathbf{A}$ exists.