
INSTRUCTIONS:

- Calculators, books, notes, and crib sheets are not permitted.
- Write your name, instructor's name, and recitation number on the front of your bluebook.
- Work all five problems. Start each problem on a new page.
- Show your work and clearly identify your final answer.

---

1. Given the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

a. Find the eigenvalues of $\mathbf{A}$ (6 points).

b. Find the eigenvectors of $\mathbf{A}$ (14 points).

2. (20 points) Solve the following initial value problem

$$y'' + \frac{1}{t}y' - \frac{4}{t^2}y = t^2 \quad y(1) = 1, y'(1) = 2, t > 0$$

using variation of parameters and knowing that the general homogeneous solution is given by

$$y_h(t) = c_1 t^2 + c_2 \frac{1}{t^2}$$

3. Consider the system of equations $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, where $\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

a. Find the characteristic equation and eigenvalues for this system (5 points).

b. Find the eigenvectors for $\mathbf{A}$ (5 points).

c. Find the general solution of this system (5 points).

d. What is the span of the eigenvectors of $\mathbf{A}$ (5 points)?

4. Given the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$$

   a. Find the eigenvalues of $\mathbf{A}$ (6 points).

   b. Find the eigenvectors of $\mathbf{A}$ (14 points).

5. For each of the following equations, determine whether the guess of the particular solution will or will not work using to the method of **undertermined coefficients** (4 points each). Note: you are *not* required to show your work.

   a. $y'' + y = 3\sin(t)$ and $y_p = c_1 \cos(t) + c_2 \sin(t)$.

   b. $y'' + y = 6\sin(2t)$ and $y_p = c_1 \cos(2t) + c_2 \sin(2t)$.

   c. $y'' - 4y' + 4y = e^{-2t}$ and $y_p = c_1 e^{-2t}$.

   d. $y'' - 4y' + 4y = te^{-2t}$ and $y_p = c_1 te^{-2t} + c_2 e^{-2t}$.

   e. $y'' - 3y' = 3t^2$ and $y_p = c_1 t^2 + c_2 t + c_3$.