
INSTRUCTIONS:

- Computers, calculators, books, notes, and crib sheets are not permitted.
- Write your name, instructor's name, and recitation number on the front of your bluebook.
- Work all **five problems**. Start each problem on a new page.
- Show your work and clearly identify your final answer.

---

1. (20 points)

   Classify each of the following differential equations by (i) order, (ii) linear or nonlinear, (iii) separable or not, and *if* it is linear (iv) homogeneous or nonhomogeneous.

   $$(a)\ \ y' + t^2 y'' = y - t^3 \qquad (b)\ \ y' = e^y t - 3 \qquad (c)\ \ y' - ty = 3y \qquad (d)\ \ y' + y'' = t\sin(y)$$

2. (20 points)

   Mark each of the following statements as True or False. (No work need be shown).

   a) $L(x, y) = 3y + 4 + x$ is a linear operator.

   b) Picard's theorem implies that $y' = \sqrt{y} + t^{1/3}$, $y(0) = 3$ has a unique solution.

   c) $y(t) = \alpha t$ is a solution of $ty' + 4y = 5t$ for $\alpha = 1$.

   d) $y(t) = \sin(t)$ is an equilibrium solution of $y' = \cos(t)$.

   e) If y_1, y_2 are solutions of $y'' + e^t y = e^{-t^2}$ then $y = y_1 - y_2$ is a solution of $y'' + e^t y = 0$.

3. (25 points)

   a) Find the homogeneous part of the general solution of $y' + ty = t$.

   b) Find a particular solution of $y' + ty = t$.

   c) Given $y(0) = 2$, what is the solution of $y' + ty = t$?

4. (15 points)

   a) Given $y'(t) = f(t, y)$, write the expression for t_{n+1} and y_{n+1} in terms of y_n, t_n, h for Euler's numerical method.

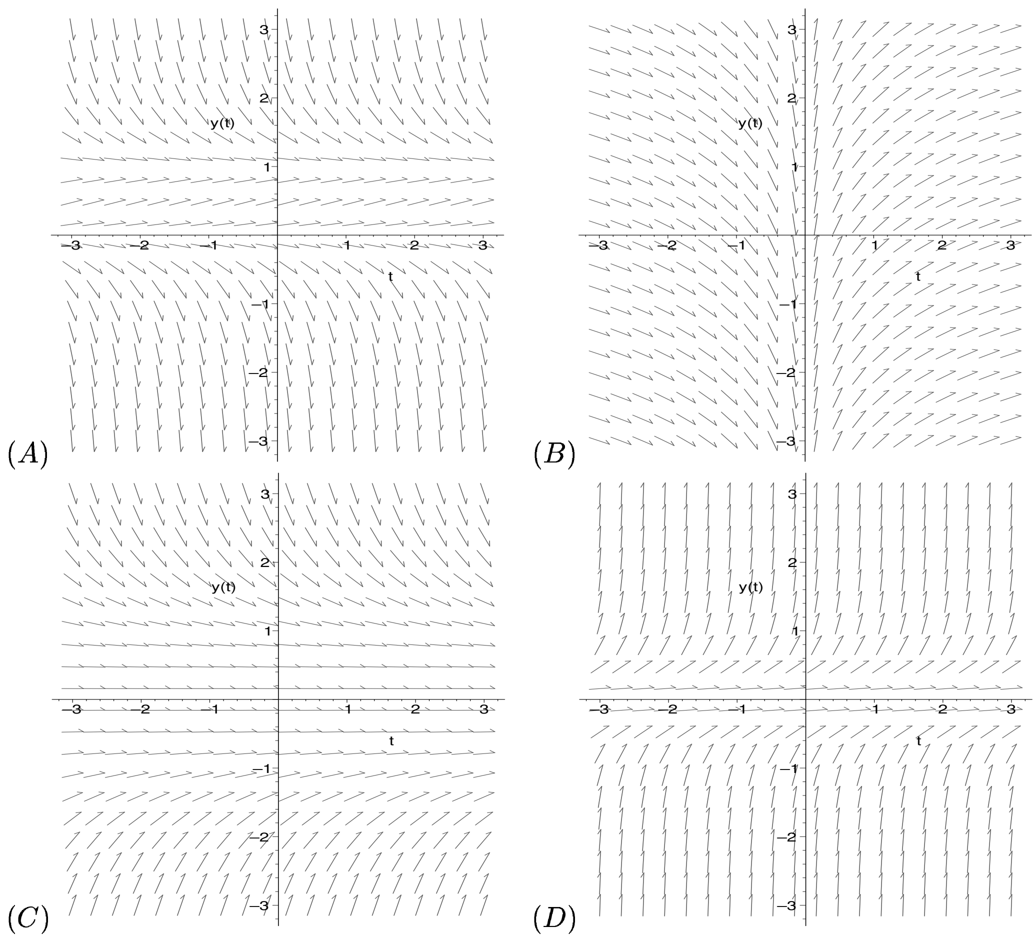
   b) Use Euler's Method to find the numerical solution for 2 steps (ie. y_1, y_2) to the problem

   $$\begin{aligned} y' &= y^2 - 6y + 5 \\ y(0) &= 3 \\ \text{using } h &= 0.25 \ \text{ as your step size.} \end{aligned}$$

5. (20 points)

a) Match each of the direction fields (A)-(D) with one of the following differential equations :

(1) $y' = \dfrac{1}{y}$ (2) $y' = y \cdot (1-y)$ (3) $y' = 3y^2$ (4) $y' + y = \sin(y)$ (5) $y' = t^3$



b) For equation (4), suppose $y(0) = 2$. What will be $\lim_{t\to\infty} y(t)$ and $\lim_{t\to -\infty} y(t)$?

c) For equation (2), what are the equilibria?

d) Classify the equilibria of equation (2) as stable, unstable or neither.