
INSTRUCTIONS:

- Computers, calculators, books, notes, and crib sheets are not permitted.
- Write your name, instructor's name, and recitation number on the front of your bluebook.
- Work all **five problems**. Start each problem on a new page.
- Show your work and clearly identify your final answer.

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1. (20 points) For the following equation

$$y'' - 4y' + 4y = \frac{e^{2t}}{t}$$

   a. Find the characteristic equation. (5 points)

   b. Find two linearly independent solutions to the homogeneous equation. (4 points)

   c. Find the particular solution. (6 points)

   d. Give the general solution. (2 points)

   e. Solve the initial value problem for $t_0 = -1$, $y(t_0) = e^{-2}$, and $y'(t_0) = e^{-2}$. (3 points)

2. (20 points)

   a. Solve the homogeneous differential equation $y'' - 2y' + y = 0$. (6 points)

   b. Find the solution to the above problem satisfying the initial value: $y(0) = 2, y'(0) = 5$. (4 points)

   c. Predict the most suitable form of y_p for the nonhomogeneous differential equation $y'' - 2y' + y = te^t$ (*DO NOT SOLVE*). (4 points)

   d. Find the particular solution to the equation in part c. (6 points)

3. For the system of linear differential equations

$$\mathbf{x}' = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$

a. Find all the eigenvalues and corresponding eigenvectors of $\mathbf{A}$. (8 points)

b. Find the general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$. (6 points)

c. Find the solution to the above problem satisfying the initial value: $x_1(0) = 4, x_2(0) = -2$. (6 points)

4. (20 points) Given the matrix

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

answer the following questions.

a. Find the eigenvalues of $\mathbf{A}$ (5 points).

b. Find the eigenvectors of $\mathbf{A}$ (10 points).

c. Give the eigenspace associated with each eigenvalue (5 points).

5. (20 points) Answer the following true/false questions:

a. If y_1 and y_2 are two solutions of $y'' - y'\sin(t) + y = 0$ then $y = c_1\,y_1 + c_2\,y_2$ is the general solution.

b. The vectors $\mathbf{u} = (1,1,1,2)$, $\mathbf{v} = (0,1,2,3)$, and $\mathbf{w} = (1,0,0,0)$ form a basis of $\mathbb{R}^4$.

c. The equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is a scalar multiple of a column in $\mathbf{A}$.

d. The solution space of $y'' + y' + y = \sin(t)$ is a 2-dimensional vector space.

e. If $\mathbf{A}\mathbf{x} = \mathbf{x}$, then $\mathbf{x}$ is an eigenvector of $\mathbf{A}$.