APPM 2360 - Exam 1 September 21, 2005 , 7 - 8.30 pm

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). Draw also a grading grid.

There are FIVE problems (with subparts a, b, ...). You must solve all five problems. Each full problem is worth 20 points. Start each problem on a new page. Show all your work in your bluebook. Explain all steps in your solutions. Box all your answers. Calculators, books or any notes are NOT permitted, with the exception of one two-sided 8½ × 11 'crib sheet' .

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1. Consider the DE $2tyy' = 1$.

   a. Draw the direction field for the DE, and sketch some examples of solution curves.

   b. Find the general solution to the DE.

2. Consider the DE $y' = \frac{y}{t} + \frac{y^2}{t^2}$.

   a. Write down the solution of the DE with initial condition $y(1) = 0$ (No calculations are required for this). Is this solution unique?

   b. Using the fact that you know one particular solution to this Riccati DE, find the solution which satisfies $y(1) = 1$.

   c. The DE is not just of Riccati type, but is also Euler-homogeneous and of Bernoulli type (so we have three ways to solve it). If we utilize that it is of Bernoulli type, write down the transformation that linearizes it. How does this compare with the transformation you used in part b?

3. In 1970 Lake Sibaya, had a salt concentration of μ kg/m^3. Fresh water enters the lake from a stream and through rainfall in the amount of α m^3 per year. The lake has no outflow, but looses water due to evaporation and domestic use. Its total volume fluctuates seasonally, i.e. its volume is a function of time $V(t)$. Evaporation does not remove any of its salt content, and domestic water use is estimated at ρ m^3 per year. The lake can be considered as well mixed at all times.

   a. Write down a mathematical model describing the total salt content of Lake Sibaya.

   b. Assuming that the total volume of Lake Sibaya remains constant, on average $V(t) = V_0$ m^3, calculate its salt content in 2005.

Please turn over ⇒

4. Consider the DE $(1+t^2)\,y' = ty + \sqrt{1+t^2}$.

a. Find the *homogeneous* solution $y_h(t)$ to the equation.

b. Find a particular solution $y_p(t)$ to the *inhomogeneous* equation, using variation of parameters.

c. Find the general solution to the DE, and give also the solution that satisfies the initial condition $y(0) = 0$.

5. Consider the DE $y' = y^2 - \varepsilon$.

a. What are the equilibrium solutions for the equation? Comment on their stability. How do they depend on ε?

b. Draw a bifurcation diagram for the equilibria locations versus ε. Mark in the diagram the stability situation along the curves.