APPM 2360

Exam 2: March 2, 2005

ON THE FRONT OF YOUR BLUEBOOK write (1) your name, (2) the name of your lecture professor and time of your lecture, (3) the name of your recitation TA and the number of your recitation, and (4) a five-problem grading grid. WHEN YOU ARE FINISHED, remember to sign the honor code pledge on the front of your bluebook. WHEN YOU TURN IN YOUR EXAM, you must show your CU ID.

Show ALL of your work in your bluebook, and **box in your final answers.** A correct answer with no relevant work shown will receive no credit. Start each problem on **the top of a new page.** This exam is closed-book and no calculators are allowed. You are allowed a one-page crib sheet. **Other than your crib sheet, no other papers, books, snacks, or devices of any sort may be on your desk.**

1. (25 points) Solve the IVP

$$x'' + x' - 2x = 3 - 2t^2, \qquad x(0) = 2,\ x'(0) = 3$$

2. (25 points) Consider the ODE $t^2y'' - 2ty' + (t^2 + 2)y = 3t^3$ (for $t > 0$).

   (a) Show that $y_1 = t\cos(t)$ and $y_2 = t\sin(t)$ are solutions of the homogeneous part.

   (b) Calculate the Wronskian $W(y_1, y_2)$.

   (c) What is the general solution of the homogeneous part? Justify your answer based on your answers to (a) and (b).

   (d) Find a particular solution. [Hint: don't forget to write the ODE in the correct form first!]

   (e) Find the general solution (in its simplest form).

3. (25 points) For those equations that can be solved using Undetermined Coefficients, write down the <u>form</u> of the particular solution. You do not need to solve for the coefficients. You do need to BOX YOUR ANSWERS:

   (a) $y'' + 4y' + 4y = 5e^t + t^2 - 1$

   (b) $y'' + 4y' + 4y = e^{-2t}$

   (c) $y'' + 4y = \sin(t) + \cos(t)$

   (d) $y'' + 4y = \sin(2t) - \cos(2t)$

   (e) $y'' + 2y' + 2y = \dfrac{t}{e^t}$

   (f) $y'' + 2y' + 2y = te^{-t}\sin(t)$

   (g) $y'' - y' = t$

   (h) $y'' - y' = 1 - \ln(t)$

   (i) $t^2y'' + y = t^2 + 1$

— OVER —

4. (10 points)

   (a) Find the general solution to the fourth order ODE $3x'''' + 6x'' + 3x = 0$
   (b) Find the general solution to the fifth order ODE $x''''' = 0$

5. (15 points) Consider the ODE $t^2y'' - 3ty' + 4y = 0$ (for $t > 0$).

   (a) By guessing a solution of the form $y = t^r$, derive the characteristic equation for r.
   (b) Show that the approach in (a) yields only one solution (y_1). However, show that $y_2 = \ln(t)y_1$ is also a solution.
   (c) Show that y_1 and y_2 are independent and, hence, determine the general solution.