APPM 2360

Exam 3: April 6, 2005

ON THE FRONT OF YOUR BLUEBOOK write (1) your name, (2) the name of your lecture professor and time of your lecture, (3) the name of your recitation TA and the number of your recitation, and (4) a five-problem grading grid. WHEN YOU ARE FINISHED, remember to sign the honor code pledge on the front of your bluebook. WHEN YOU TURN IN YOUR EXAM, you must show your CU ID.

Show ALL of your work in your bluebook, and **box in your final answers.** A correct answer with no relevant work shown will receive no credit. Start each problem on **the top of a new page.** This exam is closed-book and no calculators are allowed. You are allowed a one-page crib sheet. **Other than your crib sheet, no other papers, books, snacks, or devices of any sort may be on your desk.**

1. (15 points) Solve the system of equations

$$\begin{aligned} x + z &= 1 \\ 2x + y + z &= 3 \\ -x + 2y &= 7 \end{aligned}$$

2. (20 points) Determine the values of k for which the following linear system is consistent:

$$\begin{aligned} -kx_1 + x_2 &= k \\ (1-k)x_1 + x_2 &= 2k \\ 2kx_1 - x_2 &= 0 \end{aligned}$$

   What does a consistent system represent geometrically (in this case)?

3. (20 points) Given the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, answer the following questions:

   (a) Find the determinant of A.

   (b) For which values of θ are the rows of A linearly independent?

   (c) Find the inverse of A.

   (d) Solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$.

   (e) Does the system of equations $A\mathbf{x} = \mathbf{0}$ have a unique solution? If so, what is it?

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4. (30 points) Determine whether the following statements are true or false (not always true). You do not need to show your working/reasoning. You must write the full word TRUE or FALSE.

   (a) $(A^{-1})^{-1} = A$ (assuming A is invertible).

   (b) In a 4-dimensional vector space, the span of any three vectors forms a 3-dimensional subspace.

   (c) The system of equations $A_{m\times n}\mathbf{x}_{n\times 1} = \mathbf{0}_{m\times 1}$, when $m < n$ (*i.e.* A has more columns than rows), has infinitely many solutions.

   (d) The functions $\{1, \sin^2(t), \cos^2(t)\}$ are linearly <u>in</u>dependent.

   (e) If $AC = CB$ and $\det(C) \neq 0$, then $A = B$.

   (f) The set $\{1 + t, 2t, 1 - t^2\}$ forms a basis for P_2, the space of polynomials of degree 2 or less.

   (g) If $\det(V) \neq 0$, then the columns of V form a linearly <u>in</u>dependent set of vectors.

   (h) If $\det(M) = 0$ and $\det(B) \neq 0$, then $A = BMB^{-1}$ has no inverse.

   (i) If $A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$, then $\det(A) = 0$.

   (j) If $V = \text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ and $\mathbf{x}$ is a vector in V, then $\mathbf{x} = c_1\mathbf{v}_1 + \ldots + c_n\mathbf{v}_n$, for some constants $c_1, \ldots, c_n$.

5. (15 points) Let V be the set of all 3×3 *skew symmetric* matrices, *i.e.* matrices $A_{3\times 3}$ such that $A^T = -A$.

   (a) Is V a vector space? (Note: you may assume that the set of all 3×3 matrices is a vector space.)

   (b) What is the dimension of V?

   (c) Find a basis for V.