APPM 2360

Final Exam: May 2, 2005

ON THE FRONT OF YOUR BLUEBOOK write (1) your name, (2) the name of your lecture professor and time of your lecture, (3) the name of your recitation TA and the number of your recitation, and (4) a five-problem grading grid. WHEN YOU ARE FINISHED, remember to sign the honor code pledge on the front of your bluebook. WHEN YOU TURN IN YOUR EXAM, you must show your CU ID.

Show ALL of your work in your bluebook, and **box in your final answers.** A correct answer with no relevant work shown will receive no credit. Start each problem on **the top of a new page.** This exam is closed-book and no calculators are allowed. You are allowed a three-page crib sheet. **Other than your crib sheet, no other papers, books, flying monkeys, snacks, or devices of any sort may be on your desk.**

1. (25 points)

   (a) Find the general solution of $y'' = 2y' + 3y$

   (b) Convert the equation in (a) to a first-order system of equations and write the system in matrix-vector form

   (c) Find the eigenvalues of the matrix in (b). How do they relate to your answer to (a)?

   (d) Find the general solution to the system in (b). How does this solution — in particular, the individual components of the vector solution — relate to your answer to (a)?

2. (20 points) Find the general solution of $y'' - 4y' + 4y = f(x)$, where

   (a) $f(x) = 3e^{-x}$

   (b) $f(x) = 2\sin(2x)$

3. (30 points)

   (a) Solve the IVP system

$$\begin{cases} x' &= -4x + 3y \\ y' &= -6x + 2y \end{cases} \qquad \begin{cases} x(0) = 1 \\ y(0) = 0 \end{cases}$$

   (b) Find the general solution of

$$\begin{aligned} x' &= x + 3y \\ y' &= x - y \end{aligned}$$

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4. (20 points)

   (a) Solve the system of equations

$$\begin{aligned} x + 2z &= -2 \\ 2x - y + z &= -7 \\ -x + 2y &= 8 \end{aligned}$$

   (b) Find the determinant of the matrix that corresponds to this system.

5. (30 points) Answer the following TRUE/FALSE questions. You **MUST** write down an unabbreviated "**TRUE**" or "**FALSE**" to received credit. Note: In this problem you do not need to show your work.

   (a) The general solution of $y' = (y-t)^2$ is $y = c(1+t)$, where c is a constant.

   (b) In a 4-dimensional vector space, the span of any three vectors forms a 3-dimensional subspace.

   (c) $y' - ty = t$ is a separable equation.

   (d) A correct guess for the particular solution of $y'' - y = t\sin(t)$ is $y_p = c_1 t\cos(t) + c_2 t\sin(t)$.

   (e) The functions $\{1, \sin^2(t), \cos^2(t)\}$ are linearly dependent.

   (f) The eigenvalues of the matrices A and cA are the same for any number c.

   (g) Any nonlinear, first order, autonomous equation is separable.

   (h) The general solution of $y'' + \omega^2 y = \cos(\omega t)$ is an unbounded function.

   (i) The rows of $A(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ are linearly <u>in</u>dependent, except for $\theta = 0, \pm\pi, \pm 2\pi, \ldots$.

   (j) $ty'' + y = t + 1$ <u>cannot</u> be solved using Undetermined Coefficients.

6. (25 points) Consider the I.V.P.

$$\begin{aligned} \frac{dy}{dt} &= \frac{y^2}{y^3 - 2ty} \\ y(0) &= 1\,. \end{aligned}$$

   Solve this I.V.P. for $t(y)$ (*i.e.* treat y as the independent variable) by noting that $\frac{dt}{dy} = 1/\left(\frac{dy}{dt}\right)$.

7. (25 points) Consider the system of ODE's

$$\begin{aligned} x' &= \mu - y^2 \\ y' &= x - y \end{aligned}$$

   (a) For $\mu = -2$, find all real equilibria. Classify each equilibrium by finding the eigenvalues of the linearized ODE's around each equilibria.

   (b) Repeat for $\mu = 1$. Sketch the phase portrait of the (non-linear) system in this case.

   (c) Define the bifurcation value, μ_0, as the value of μ where the number of equilibria changes. What is μ_0 for this system?

   (d) Draw a bifurcation diagram for this system by plotting the y location of the equilibria as a function of μ. Be sure to label the stability of each equilibria on this plot.

8. (25 points) Match the following systems with the correct phase portrait/vector field. Note: In this problem you do not need to show your work.

(i) $\begin{cases} x' = x - xy \\ y' = -y + xy \end{cases}$ (iv) $\begin{cases} x' = y - y^3 \\ y' = x \end{cases}$

(ii) $\begin{cases} x' = x - xy \\ y' = y - xy \end{cases}$ (v) $\begin{cases} x' = -y + y^3 \\ y' = x \end{cases}$

(iii) $\begin{cases} x' = xy \\ y' = y - x^2 + 1 \end{cases}$