

**INSTRUCTIONS:**

- Computers, calculators, books and notes are not permitted.
  - An 8.5x11 inch crib sheet is allowed.
  - Write your name and instructor's name on the front of the test.
  - Show your work and clearly identify your final answer.
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1. (15 points)

For the following IVP

$$\frac{dy}{dt} = 3y^{2/3} \quad y(2) = 0,$$

- (a) verify that  $y_1(t) = 0$  is a solution.
- (b) Verify that  $y_2(t) = (t - 2)^3$  is also a solution.
- (c) Does this violate Picard's Theorem? Explain.

2. (30 points)

(a) Solve the following differential equation

$$y' = -\cos(t)(y^2 - 2y).$$

(b) Solve the following IVP

$$y' + \frac{1}{t+1}y = 2e^{3t}, \quad y(0) = 1.$$

3. (27 points)

Our job is to dilute 100 gals. of salt water in a tank. Fresh water is poured at a rate of 2 gal/min into the tank that has an initial concentration of 1 lb/gal. The stirred mixture is drained out of the tank at a rate of 2 gal/min.

- (a) Find the amount of salt in the tank at anytime.
- (b) Suppose the outflow drain ruptures at 50 mins, doubling the outflow rate. Find the amount of salt in the tank since the rupture.
- (c) How long will it take after the rupture for the tank to be empty?

4. (18 points)

For the following  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are  $n \times n$  matrices. Are the following statements necessarily true, *yes* or *no*? Give a short reason why or why not.

- (a) The linear system  $\mathbf{Ax} = \mathbf{0}$  has the unique solution  $\mathbf{x} = \mathbf{0}$ , where  $\mathbf{x}$  and  $\mathbf{0}$  are  $n \times 1$  matrices.
- (b) If the RREF of an augmented matrix has a row of the form  $[0 \ 0 \ \cdots \ 0 | k]$  then the associated system of equations is inconsistent.
- (c) If  $\mathbf{AB} = \mathbf{AC}$  then  $\mathbf{B} = \mathbf{C}$ .
- (d) If  $\mathbf{A}$  is invertible then  $(\mathbf{ABA}^{-1})^3 = \mathbf{AB}^3\mathbf{A}^{-1}$ .
- (e) If  $|\mathbf{A}||\mathbf{B}| = 0$  then  $(\mathbf{AB})^{-1}$  does not exist.
- (f) If  $|\mathbf{A}||\mathbf{B}| = |\mathbf{B}||\mathbf{A}|$  then  $\mathbf{AB} = \mathbf{BA}$ .

5. (10 points)

Find a value of  $k$  to make

$$\begin{aligned}x + 3y &= 2 \\ -2x + ky &= 1\end{aligned}$$

an inconsistent system of equations.

6. (25 points)

For the following system

$$\begin{aligned}\frac{dx}{dt} &= 1 - x - y \\ \frac{dy}{dt} &= x - y^2 + 1,\end{aligned}$$

(a) Determine and plot the equilibrium points and nullclines of the system.

(b) What is the slope of the solution at the point  $(1, 1)$ ?

7. (25 points)

(a) Is the collection of all solutions to the following differential equation

$$y' + (e^t + 4)y - 2t + 6 = 0$$

a vector space? Explain.

(b) What is a basis of the subspace of  $\mathbb{R}^4$  defined by the equation,

$$x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

Show that the basis vectors are linearly independent.