

INSTRUCTIONS:

- Computers, calculators, books and notes are not permitted.
 - An 8.5x11 inch crib sheet is allowed.
 - Write your name and instructor's name on the front of the test.
 - Show your work and clearly identify your final answer.
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1. (15 points)

Solve the following differential equation

$$y' = (y^2 - 1) \sin(t).$$

2. (20 points)

Solve the following differential equation

$$y'' + 2y' + y = 4e^{-t}.$$

3. (15 points)

Solving the following differential equation

$$y' + \frac{3}{t}y = t^{-2} \sin(t).$$

4. (20 points)

Solve the following IVP

$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

5. (30 points)

For the following system

$$\begin{aligned} x' &= y - 4x \\ y' &= x^2 + c - y, \end{aligned}$$

- Let $c = 1$. Sketch nullclines and identify equilibrium points.
 - Let $c = 0$. (1) Sketch nullclines and identify equilibrium points. (2) Determine the stability of the fixed point(s) by calculation of the Jacobian.
 - What is a value of c so that the system has no real fixed points?
6. (20 points)

For the following matrix

$$\mathbf{A} = \begin{bmatrix} 5 & -6 \\ 3 & -1 \end{bmatrix},$$

- find its eigenvalues and eigenvectors. (b) State the eigenspace for each eigenvalue.

7. (15 points)

(a) Let \mathbb{Q}_n be the set of all odd polynomials of degree less or equal to n , which includes the zero polynomial. Odd polynomials are polynomials that have terms of only odd powers. For $n = 5$, is \mathbb{Q}_n a vector space? If yes, what is a basis and what is its dimension? If it is not a vector space, what will make it a vector space?

(b) Is \mathbb{Q}_n a subspace of \mathbb{P}_n , set of all polynomials of degree less or equal to n .

8. (15 points)

Write the following system of second order equations

$$\begin{aligned}x'' + 3x' - 2x - y &= \cos(3t), \\y'' - 2x + y - y' &= \sin(t),\end{aligned}$$

as a system of first order equations in the form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}(t)$.