APPM 2360–Exam 2 **October 19, 2005, 7–8.30pm**

Instructions: On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). Draw also a grading grid.

There are FIVE problems (with subparts a, b, ...). You must solve all five problems. Each full problem is worth 20 points. Start each problem on a new page. Show all your work in your bluebook. Explain all steps in your solutions. Box all your answers. Calculators, books or any notes are NOT permitted, with the exception of one two-sided $8\frac{1}{2} \times 11$ 'crib sheet'.

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1. (a) Given

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$$

calculate AB.

(b) Calculate $(AB)^T$ by multiplying A^T and B^T where A and B are given in question 1a. Verify your answer by also taking the transpose of the product AB you calculated in question 1a.

(c) Given

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$$

calculate AB and BA. If you are unable to do the multiplication, give the reasons.

2. (a) Solve the following system of equations for x, y and z by using row reductions (you need not do a full reduction, i.e. ones on the diagonal),

$$\begin{aligned} x + y + z &= 1 \\ 2x - y + z &= 4 \\ x + y - z &= -1. \end{aligned}$$

(b) By using row reductions, find for which value(s) of λ does the system

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ \lambda \end{bmatrix}$$

has a solution(s). How many solutions are there for this value of λ? Explain in detail.

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3. (a) Calculate the determinants of the following matrices,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

You are allowed to use your calculations of question 2.

(b) Find a basis for the column space of A in question 3a.

(c) Which of the following form a vector space. Give a full explanation.

i. All $n \times n$ matrices A with $\det(A) = 1$.

ii. All $n \times n$ matrices A such that $\text{trace}(A) = 1$.

4. Find the general solutions of the following equations. Draw rough graphs to illustrate the nature of the solutions

(a) $y'' - y' - 2y = 0$

(b) $4y'' + 12y' + 9y = 0$

5. Consider the system of differential equations

$$\begin{aligned} x' &= y \\ y' &= -y - x + x^3 \end{aligned}$$

(a) Calculate and draw the isoclines of the system. Show their directions.

(b) Calculate the equilibrium points and indicate them on your drawing.