APPM 2360 **Exam 3** **November 16 2005, 7 – 8:30pm**

Instructions: On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). Also draw a grading grid.

There are FIVE problems (with subparts a, b, ...). You must solve all five problems. Each full problem is worth 20 points. Start each problem on a new page. Show all your work in your bluebook. Explain all steps in your solutions. Box all your answers. Calculators, books or any notes are NOT permitted, with the exception of one two-sided $8^1\!/_2 \times 11$ 'crib sheet'.

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1. Consider the second order differential equation:

$$y'' + 2y' + 2y = \sin(t)$$

   (a) Write the characteristic equation and solve the associated homogeneous problem.

   (b) Using the method of undetermined coefficients find a particular solution to the inhomogenous problem.

   (c) With the initial conditions $y(0) = 0$, $y'(0) = 0$ calculate the complete solution to the system.

2. For the differential equation with parameter $b \in \mathbb{R}$,

$$y'' + 2by' + 4y = 4$$

   (a) What is the homogeneous solution for $b \neq 2$?

   (b) For what values of b does *this* homogeneous solution decay to zero as $t \to \infty$?

   (c) Taking $b = 2$ find the homogeneous solution.

   (d) Find the particular solution for $b = 2$ using variation of parameters.

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3. (a) Put the complex number $z = \sqrt{2} + \sqrt{2}i$ into polar form, i.e. find r and θ so that $z = re^{i\theta}$.

   (b) Find *all* complex numbers $z = a + ib$ such that $e^z = 3$.

   (c) Calculate the imaginary part of $\frac{1}{2+2i}$

   (d) Using complex exponentials calculate $\int e^{2x} \sin(2x) dx$.

4. Given the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 4 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

   (a) Given that the matrix A has zero determinant, namely $\det(A) = 0$. What is one eigenvalue of this a matrix?

   (b) Find the other two eigenvalues of A.

   (c) Find ONE eigenvector of A.

5. For the system of linear differential equations given by:

$$\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \mathbf{f}(t),$$

   (a) Find the solution to the associated homogeneous system.

   (b) Find a particular solution for the system with the forcing function,

$$\mathbf{f}(t) = \begin{bmatrix} e^t \\ 1 \end{bmatrix}$$