APPM 2360 - Final December 9, 2005 , 7.30 - 10.00 pm

On the front of your blue book, as well as in the space below on this problem set, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). On the front of the blue book, draw also a grading grid.

Your name: ______________________________

Lecturer's name: (Docherty, Fornberg, or Herbst): ____________________

TA name, or Recitation session number: ____________________

There are EIGHT problems (with subparts a, b, ...). You must solve all eight problems. Each full problem is worth 25 points. For Problems 1-6, you must explain all the steps in your solutions. For Problems 7 and 8, you need only to give answers - no explanations are needed. Box all your answers. Calculators, books or any notes are not permitted, with the exception of up to three two-sided 8½ × 11 'crib sheets' .

For Problem 8, fill in your answer on your copy of this problem set, and return this whole set with your blue book.

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1. Calculate the solutions to the following initial value problems

   a. $y' = y^2,\ y(0) = 1$. At what time does the solution become infinite?

   b. $y' = \frac{y}{t} + 1,\ y(1) = 1$.

   c. $y' = \frac{y}{t} + y^3,\ y(1) = \sqrt{3}$.

2. When you pour your tea, its temperature is $90^0\ C$, and the room temperature is $30^0\ C$. An hour later your tea's temperature is $60^0\ C$.

   a. Write down the differential equation that describes the change in temperature of your tea.

   b. Solve the initial value problem to obtain the temperature of your tea at any time.

3. a. Solve the following system of equations for x, y and z

$$\begin{cases} x+y+z = 1 \\ x-y+z = -1 \\ x+y-z = 3 \end{cases}$$

b. Calculate the determinant of the coefficient matrix in the system above.

c. Calculate the dimension and determine a basis for the column space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 3 & -1 & 1 & -1 \end{bmatrix}.$$

4. Consider the matrix $A = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

a. Given that $\underline{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector, calculate the corresponding eigenvalue.

b. Calculate two independent eigenvectors belonging to the eigenvalue $\lambda = 1$.

c. Find the value of det(A), using only the results from part a and/or b above.

5. Find one particular solution to the DE $y'' - y = e^{-2t}$ by means of

a. Method of undetermined coefficients,

b. Variation of parameters.

6. Consider the autonomous system of DEs

$$\begin{cases} x' &= (x^2-1)\,y \\ y' &= (1-y^2)(x+\frac{3}{10}y) \end{cases}$$

a. Determine all the equilibrium points.

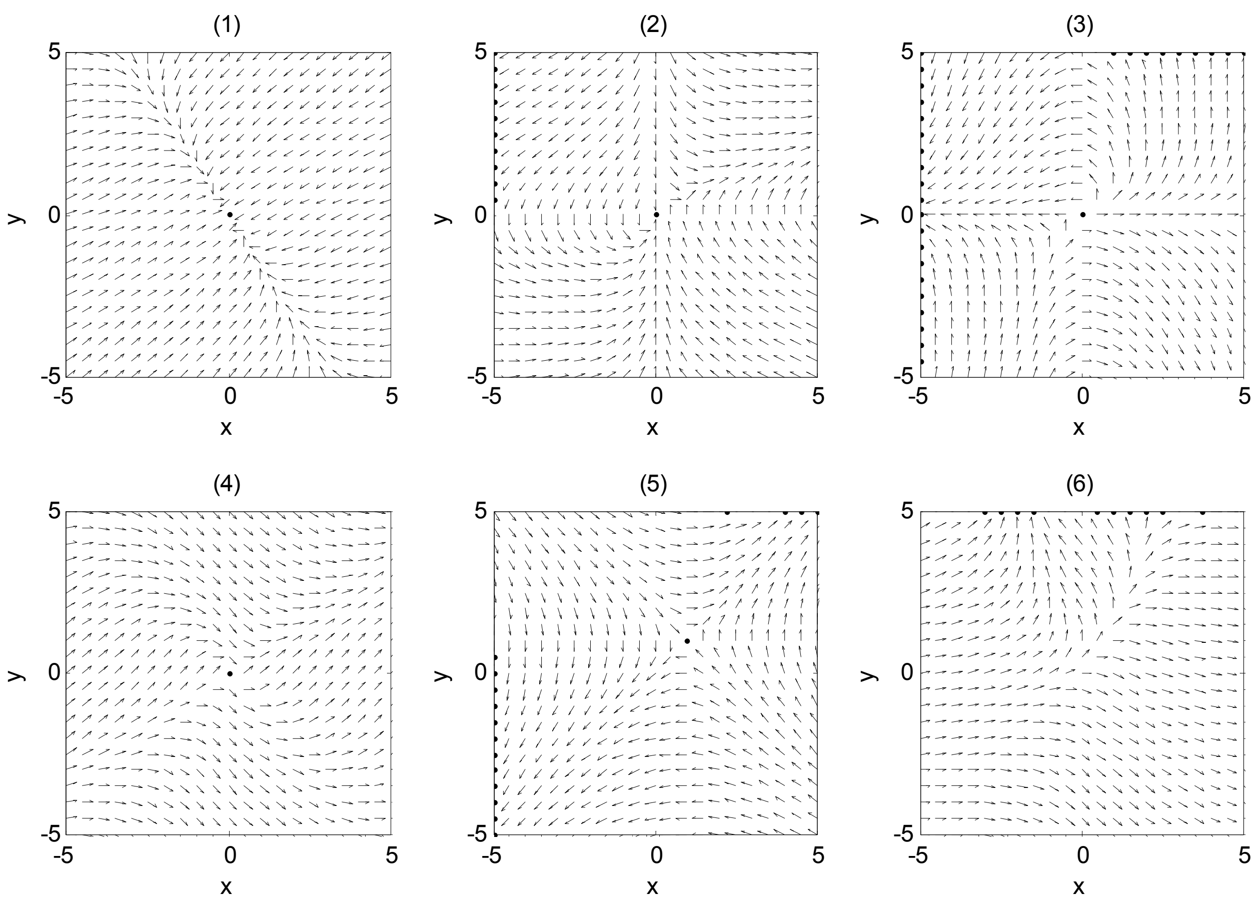
b. One of the equilibrium points is located at the origin, and it is a spiral. Determine if it is a spiral in or spiral out. Also determine if the solution trajectory for this spiral goes around in clockwise or anti-clockwise direction in the (x,y)-plane.

7. Consider the following six autonomous systems of ODEs:

a. $\begin{cases} x' &= y-1 \\ y' &= x-1 \end{cases}$ b. $\begin{cases} x' &= x^2+y^2 \\ y' &= x^2-y^2 \end{cases}$ c. $\begin{cases} x' &= xy \\ y' &= x-y \end{cases}$

d. $\begin{cases} x' &= x-y \\ y' &= xy \end{cases}$ e. $\begin{cases} x' &= x^2-y+1 \\ y' &= y-x \end{cases}$ f. $\begin{cases} x' &= -2x-y \\ y' &= -x-y \end{cases}$

Match each system with its phase plane picture below: (For this problem, no explanations are needed; just give your answers in the blue book)



8. Multiple choice - for each question, mark below by a cross either True or False (i.e. not always correct). Each of the 20 questions is worth 1 point, and there is a 5 point bonus if you get them all correct. You need not give any explanations.

		True	False
a.	The DE $y' = t + y$ is separable	☐	☐
b.	The DE $y' + \sqrt{t}\, y = e^{-3t}$ is linear	☐	☐
c.	The DE $y' - y = y^3$ is a Bernoulli equation	☐	☐
d.	If the rows of a matrix are linearly independent, so are the columns	☐	☐
e.	It holds that $\dfrac{a+ib}{c+id} = \dfrac{(ac+bd)-i(bc+ad)}{c^2+d^2}$	☐	☐
f.	A quadratic equation $x^2 + ax + b = 0$ with real coefficients can have a double root that is not real	☐	☐
g.	If the eigenvalues to a real matrix are complex, so are the eigenvectors	☐	☐
h.	The system $\begin{cases} x' &= xy \\ y' &= 1 - x^2 - y^2 \end{cases}$ has five equilibrium points	☐	☐
i.	The matrix $\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$ has two independent eigenvectors	☐	☐
j.	The matrix in problem 8i above is singular	☐	☐
k.	All solutions to $y'' + 4y = \cos 2t$ grow unbounded with time	☐	☐
l.	$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 1$	☐	☐
m.	If A and B are square matrices, then $(AB)^T = A^T B^T$	☐	☐
n.	If $\lvert b \rvert > 1$, then solutions to $y' - by = 0$ grow with time	☐	☐
o.	The solutions to $y' = -t/y$ form circles in the (t,y)-plane	☐	☐
p.	The integrating factor for $y' + y = \dfrac{1}{1+e^t}$ is e^t	☐	☐
q.	$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$	☐	☐
r.	If A is invertible (non-singular), the linear system $Ax = b$ has a unique solution	☐	☐
s.	Vectors that form a base are always linearly independent	☐	☐
t.	$\begin{bmatrix} 1 \\ i \end{bmatrix}$ is an eigenvector to the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	☐	☐