APPM 2360–EXAM II **7.00-8.30pm March 15th 2006**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). Draw also a grading grid. There are FIVE problems (with subparts a, b, ...). You must solve all five problems. Each full problem is worth 20 points. Start each problem on a new page. Show all your work in your bluebook. Explain all steps in your solutions. Box all your answers. Calculators, books or any notes are NOT permitted, with the exception of one two-sided $8.5'' \times 11''$ 'crib sheet'

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1. a. Convert the linear system

$$\begin{aligned} z + 3x &= 1 \\ 2y - x + 2z &= -1 \\ -z + 5x - 3y &= 1 \end{aligned}$$

into matrix-vector form $\mathbf{Ax} = \mathbf{b}$. Identify $\mathbf{A}, \mathbf{b}$.

b. From your answer in a. calculate $\mathbf{b}^T\mathbf{A}^T$ and verify that it is identical to $(\mathbf{Ab})^T$.

c. Which of the following matrix mutiplication exists: (i) $\mathbf{A}^T\mathbf{b}$ (ii) $\mathbf{Ab}^T$ (iii) $\mathbf{b}^T\mathbf{A}$.

2. a. If possible, use Gaussian elimination to solve the following system of linear equations for x, y, and z:

$$\begin{aligned} 3x + z &= 1 \\ -x + 2y + 2z &= -1 \\ 5x - 3y - z &= 1 \end{aligned}$$

If it is not possible, give a reason.

b. Solve the matrix, vector equation

$$\mathbf{Ax} = \mathbf{b}$$

where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & \frac{1}{2} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}.$$

3. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 16 & 0 & 4 & 7 \\ -3 & 8 & 8 & 2 \\ 1 & 0 & 5 & 2 \\ -7 & 6 & 5 & 4 \end{bmatrix}$$

   a. Does the matrix $\mathbf{A}$ have an inverse ?

   b. Write down the RREF of $\mathbf{A}$ without performing any computations.

   c. Characterize the solutions of $\mathbf{A}\mathbf{x} = \mathbf{0}$.

   d. Characterize the solutions of $\mathbf{A}\mathbf{x} = \mathbf{b}$.

4. Given the vectors:

$$\mathbf{y}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{y}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

   a. Are these vectors linearly independent ?

   b. Define the two properties of a basis. Do these vectors form a basis ?

   c. Find three coefficients a_i such that

$$\mathbf{z} = \begin{bmatrix} 4 \\ -8 \\ 1 \end{bmatrix} = a_1\mathbf{y}_1 + a_2\mathbf{y}_2 + a_3\mathbf{y}_3.$$

5. Consider the second-order ordinary differential equation (ODE):

$$y'' - 2y' + y = 0$$

   a. Find two linearly independent solutions $y_1(t)$ and $y_2(t)$.

   b. Calculate the Wronskian of $y_1(t)$ and $y_2(t)$. What is the dimension of the vector space spanned by $y_1(t)$ and $y_2(t)$?

   c. Using $y_1(t)$ and $y_2(t)$ construct the general solutions $y(t)$ to the ODE.

   d. Find the solution with the conditions $y(0) = 1$ and $y(1) = 0$.