APPM 2360–EXAM III **7.00-8.30pm April 19th 2006**

On the front of your blue book, write your name, the names of your lecturer (or lecture session number) and of your TA (or recitation section number). Draw also a grading grid. There are FIVE problems (with subparts a, b, ...). You must solve all five problems. Each full problem is worth 20 points. Start each problem on a new page. Show all your work in your bluebook. Explain all steps in your solutions. Box all your answers. Calculators, books or any notes are NOT permitted, with the exception of one two-sided $8.5'' \times 11''$ 'crib sheet'

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1. For each of the following equations write down the form of the particular solution according to the method of undetermined coefficients **if it is possible to use this method** (you do NOT need to find the values of the coefficients). If it is not possible to use the method of undetermined coefficients, state another method that could be used to solve for the particular solution (you do NOT need to find the particular solution).

   a) $y'' + 3y' + 2y = t^2$

   b) $y'' + 3y' + 2y = \frac{1}{3}e^{-t}$

   c) $y'' + 3y' + 2y = te^t$

   d) $y'' + y = \tan(t)$

2. a) Find the general solution to the following non-homogeneous ordinary differential equation (ODE), using variation of parameters to find the particular solution.

   $$y''(t) + 5y'(t) + 4y(t) = \sinh(t)$$

   Where $\sinh(t) = \frac{1}{2}\left(e^t - e^{-t}\right)$.

   b) For the initial conditions $y(0) = \frac{1}{5}$, $y'(0) = \frac{1}{30}$ find the specific solution to the ODE in part (a).

3. Consider the matrix

   $$\mathbf{A} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

   a) Find the eigenvalues and eigenvectors of this matrix.

   b) What is the dimension of the eigenspace for each eigenvalue ?

4. Consider the linear system of equations $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Find the homogeneous solution $\mathbf{x}(t)$.

b) Sketch the phase plane diagram for this problem.

c) The solution of $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with initial condition $\mathbf{x}(0) = \mathbf{x}_0$ is

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}_0$$

Find the solution $\mathbf{x}(t)$, and the matrix exponential $e^{\mathbf{A}t}$, given the initial condition

$$\mathbf{x}(0) = \mathbf{x}_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Note $e^{\mathbf{A}t} = \mathbf{X}(t)\mathbf{X}^{-1}(0)$ where $\mathbf{X}(t)$ is the fundamental matrix of solutions at time t.

5. Consider the following initial value problem:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \text{where} \quad \mathbf{A} = \begin{pmatrix} \frac{4}{3} & -\frac{5}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{pmatrix}$$

a) Find the eigenvalues of the matrix $\mathbf{A}$.

b) Find the eigenvectors of the matrix $\mathbf{A}$.

c) Construct two real-valued, linear independent solutions to the system.

d) From your answer to c) solve the initial value problem.